

Does the Universe Have a Static Future? A Theoretical Evaluation Based on Free-Fall Expansion and Equivalent Time Dilations

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Abstract

The title question is meant literally: we ask whether the expansion ends in a static universe, and derive the expansion history from the velocity of free fall instead of from an expansion of space.

Following RICHARD FEYNMAN's field-dynamic treatment of gravitation, the expansion is described as an outward free fall, which removes the conflict between a peculiar velocity of the celestial bodies and a comoving frame. Two elements carry the argument. The first is the equality of kinetic and gravitational time dilation: the same quantity may be read as a velocity or as a potential, so that the kinetic and the gravitational view are interchangeable throughout. The second is the vanishing of the total energy in free fall. Matter is taken to be baryonic, its dark component arising through a mass factor q and dark energy through a factor x . Because the two dilations agree, q can be written without any motion at all, as $q = 1/(1 + \Phi/c^2)$: the mass factor is a property of the potential.

The algorithm is closed by Newton iteration and yields a present cosmic radius $R_0 = 1.2775 \cdot 10^{26}$ m and a total baryonic mass $M = 3.696 \cdot 10^{51}$ kg. After the Big Bang the model passes through a maximum of about 111 % of the present size and then contracts towards a static universe of ~ 97 %, reached after roughly five times the present age. Spacetime is curved: hyperbolic at the Big Bang, briefly spherical, slightly hyperbolic today; the measured $\Omega_k = 0.0007$ is an input. The Hubble tension is addressed by assigning the two measured values to distinct quantities, a geometric H_{CMB} and an observed $H_{\text{DL}} = c/R$. The latter falls from 72.4 to about $65 \frac{\text{km}}{\text{s} \cdot \text{Mpc}}$ at $a \approx 0.15$ and then rises steeply — a non-monotonic behaviour that Λ CDM does not show and that a sufficiently deep distance ladder can falsify.

Keywords: dark matter, dark energy, Hubble tension, general relativity, cosmology, static universe, DESI, quintessence

1 Introduction

At the core of the present picture stands the postulate that, alongside dark energy, there is only baryonic matter. Matter is not the sum of dark and baryonic matter; it is the relativistic mass of baryonic matter in the sense of ALBERT EINSTEIN's theories of relativity.

In the standard picture the expansion of the universe is the expansion of space itself: space carries the celestial bodies along, and they acquire no velocity relative to it. Drawing instead on RICHARD FEYNMAN's treatment of gravitation as a field [1], we postulate space and gravitation as a field-dynamic phenomenon, which permits us to speak of a genuine velocity.

On this view the expansion is not the aftermath of a chaotic explosion of all matter, but the result of a negative gravitation driven by potential and dark energy. The expansion velocity is accordingly the outward velocity of free fall: the free-fall velocity at the boundary of the sphere, that is the potential equivalent v_ϕ of Eq. (3), and not the rate dR/dt at which the cosmic radius itself changes.

Two readings run through this article, and it is worth keeping them apart from the outset. In the *kinetic* reading the expansion is a real outward motion: there is a velocity, and with it the time dilation that belongs to a moving clock. In the *static* reading the same state of affairs is described gravitationally, through the potential and the time dilation that belongs to a clock in a field. Equation (1) states that the two dilations are equal, and it is this equality alone that licenses the article to pass from one reading to the other. Neither is the more fundamental; which one is used is a matter of what is being calculated. Because the two readings describe the same thing in different words, the central quantity carries two names as well: kinetically it is the free-fall velocity v , gravitationally the potential equivalent v_ϕ of Eq. (3). They are the same number throughout, and the name is chosen to match the reading in force.

The article is arranged accordingly. Section 2 develops the foundations kinetically, from the velocity of free fall, with the exception of Sect. 2.5, which takes the static limit to the solar system. Section 3 is static: for a static field it shows that the equality of the two time dilations is energy conservation. Section 4 carries both readings at once — it is where the transition is made — and within it Sect. 4.1 argues statically, from the mass enclosed by a sphere. Section 5 is kinetic again: it treats the five unknown functions of the master equation, first using present-day values alone and then following the trajectory towards the Big Bang by means of the internal relativistic relation. Section 6 presents the numerical results, where both readings stand side by side, while Sect. 6.2 is static throughout, the redshift being a property of the line of sight rather than of a motion. Section 7 and Sect. 7.1 are static as well: they turn to the spatial curvature, which emerges here as a property of the model rather than as an assumption, to the end state towards which the trajectory runs, and to the explanations the model suggests for current problems in cosmology. Section 8 closes with a description of how the model works, in both readings.

2 Foundations

This section argues kinetically: everything in it is developed from the velocity of free fall. The Λ CDM standard model enters the present theory only in that the final statements must be comparable with it. Since even derivations from fundamental physics are reinterpreted in what follows, we set out the foundations in some detail.

2.1 Free Fall

The velocity of free fall from infinity is derived here not from gravity as such, but from the equality of kinetic (SR) and gravitational (GR) time dilation:

$$\sqrt{1 - \frac{v^2}{c^2}} = 1 + \frac{\phi}{c^2} \quad (1)$$

Under this condition, the velocity v is:

$$v = \sqrt{2 \frac{MG}{R} - \left(\frac{MG}{Rc} \right)^2} \quad (2)$$

Equation (1) may equally be read from the other side. Solved for v without the detour through motion, the same number stands as an expression of the potential alone. We call it the *potential equivalent* and write it v_ϕ :

$$v_\phi \equiv c \sqrt{1 - \left(1 + \frac{\phi}{c^2} \right)^2} = c \sqrt{2u - u^2}, \quad u \equiv -\frac{\phi}{c^2} \quad (3)$$

With $\phi = -\frac{MG}{R}$ this is Eq. (2) word for word. Hence v and v_ϕ are not two quantities but one, read kinetically in the first case and gravitationally in the second, and Eq. (1) is precisely what permits the exchange. Which of the two names this article uses is governed by the reading of the section in question: where the argument runs on motion we write the free-fall velocity v , and where it runs on the potential we write the potential equivalent v_ϕ .

General relativity provides the check. There the gravitational time dilation reads $\sqrt{1 - 2 \frac{MG}{Rc^2}}$, so that under the condition of equivalent time dilations one obtains the well-known expression for the second cosmic velocity, the escape velocity $v_e = \sqrt{2 \frac{MG}{R}}$, which corresponds to the free-fall velocity v when start and target heights are exchanged. It is not a third name for the same number: v_e follows from the general-relativistic form of the time dilation and v from FEYNMAN'S, and the two differ by the quadratic term of Eq. (2).

The gravitational time dilation $1 - \frac{MG}{Rc^2}$ is based on an article by RICHARD FEYNMAN [2], written in 1964. There is no contradiction to GR herein.

2.2 Gravitation

The core of the derivation of gravitational acceleration from v , using the example of free fall, is the gravitational potential:

$$v^2 = 2 \int_R^\infty g(r) dr \quad (4)$$

$$g = \frac{d}{dR} \left(-\frac{MG}{R} + \frac{1}{2} \frac{M^2 G^2}{R^2 c^2} \right) \quad (5)$$

$$g = \frac{MG}{R^2} - \frac{M^2 G^2}{R^3 c^2} \quad (6)$$

The term $-\frac{M^2 G^2}{R^3 c^2}$ appears in GR, where it is called the perturbation parameter [3], because EINSTEIN ignored it due to its obvious negligibility. We shall later learn of its elementary significance for this theory. The effect of the perturbation parameter with respect to gravitational time dilation is also described by FEYNMAN in the form:

$$1 - \frac{GM}{rc^2} \rightarrow \dots \rightarrow \sqrt{1 - \frac{2GM}{rc^2}} \quad (7)$$

2.3 Energy of Free Fall

Following NEWTON, the difference of gravitational potentials is formed by subtracting the initial from the final state. Reversing the order yields precisely the released energy that is converted into kinetic energy:

$$E_g = m(\phi_R - \phi_r) \quad (8)$$

The other way around, it would be the escape velocity — the vertical launch velocity to reach height R — which would then take on complex values.

2.4 Force from the Speed of Light in a Gravitational Field

Due to time dilation, the speed of light is also affected by the gravitational field:

$$c' = c \left(1 - \frac{MG}{Rc^2} \right) \quad (9)$$

The relation itself is not new; what appears to be unusual is to write it as a difference of energies, in the following way:

$$\Delta E_0 = m((c')^2 - c^2) \quad (10)$$

This ΔE_0 agrees with the energy of free fall in the following way: according to Newton, the approach is $E_{kin} + E_{pot} = \text{const.}$ From the previous section we know that $E_g = E_{kin}$. Now, however, $E_g + E_{kin} = -\Delta E_0$. Since $c' < c$, the quantity ΔE_0 is negative, whereas the kinetic energy is positive. Thus:

$$E_{kin} = E_g = -\frac{1}{2} \Delta E_0 = \frac{1}{2} m(c^2 - (c')^2) \quad (11)$$

The quotient $\frac{\Delta E_0}{2m}$ is then the gravitational potential, which leads to the gravitational acceleration g :

$$g = \frac{d}{dR} \frac{1}{2} \left(c^2 \left(1 - \frac{MG}{Rc^2} \right)^2 - c^2 \right) = \frac{MG}{R^2} - \frac{M^2 G^2}{R^3 c^2} \quad (12)$$

Together with the perturbation parameter, this g is already known from the considerations on gravitation. Where the expression comes from is settled in Sect. 2.5: $\Phi(R) = \frac{1}{2}((c')^2 - c^2)$ is the weak-field expansion of the SCHWARZSCHILD potential, and the second term of g is the next term of EINSTEIN's law rather than an addition to NEWTON's.

2.5 The Weak-Field Potential and the Solar System

The acceleration g derived above is not an isolated construction: it is precisely the weak-field expansion of the SCHWARZSCHILD potential [4]. This deserves to be made explicit, because it fixes the relation of the present model to general relativity in the one regime where the latter is tested to high precision.

Here the static reading is taken up for the first time. SCHWARZSCHILD's exact solution for the static, spherically symmetric field of a mass M has the metric coefficient

$$g_{00} = 1 - \frac{2MG}{Rc^2}, \quad (13)$$

so that a clock at rest at radius R runs slow by the factor $\sqrt{g_{00}}$. This is exactly the expression to which FEYNMAN's form converges in Eq. (7), and it is what the postulate of equivalent time dilations asserts; Sect. 3 shows where that equality comes from.

Taking the speed of light in the field from Eq. (9), $c' = c \left(1 - \frac{MG}{Rc^2} \right)$, the potential per unit mass follows as

$$\Phi(R) = \frac{1}{2} ((c')^2 - c^2) = -\frac{MG}{R} + \frac{1}{2} \frac{M^2 G^2}{R^2 c^2}, \quad (14)$$

whose radial derivative returns Eq. (12) without further assumptions. Comparing

$$\left(\frac{c'}{c} \right)^2 = 1 - \frac{2MG}{Rc^2} + \left(\frac{MG}{Rc^2} \right)^2 = g_{00} + \left(\frac{MG}{Rc^2} \right)^2 \quad (15)$$

shows where the perturbation parameter comes from: the first-order form $1 - \frac{MG}{Rc^2}$ and the exact coefficient g_{00} agree to first order and separate at second order by precisely that amount. The term $-\frac{M^2 G^2}{R^3 c^2}$ is therefore not an addition to NEWTON's law but the next term of EINSTEIN's.

Why the perihelion does not follow from g

It must be emphasised that g is the radial acceleration of a body *at rest*. It is not an equation of motion, and the perihelion advance cannot be obtained from it. The orbit follows from the geodesic equation, which for the SCHWARZSCHILD metric takes the form

$$\frac{d^2 u}{d\varphi^2} + u = \frac{MG}{L^2} + \frac{3MG}{c^2} u^2, \quad u = \frac{1}{r}, \quad (16)$$

where L is the angular momentum per unit mass. The final term is absent from any purely Newtonian force law; integrating it over one revolution gives EINSTEIN's result [5]

$$\Delta\varphi = \frac{6\pi MG}{c^2 a (1 - e^2)}, \quad (17)$$

which for Mercury amounts to $42.98''$ per century, confirmed by ranging to the MESSENGER spacecraft as $42.9799 \pm 0.0009''$ per century [6]. Decisively, the term $\frac{3MG}{c^2}u^2$ carries the angular momentum through u ; the spatial part of the metric contributes as much to it as g_{00} does.

Why this is not a scalar theory

This distinction separates the present approach from a scalar theory of gravitation. If a gravitational potential feeds g_{00} alone and the spatial geometry is left flat, as in NORDSTRÖM's theory [7], the resulting orbit equation loses the u^2 term and retains only a $1/r^2$ perturbation of the potential. The apsidal angle is then $\pi/\sqrt{1-\beta}$ with $\beta = \left(\frac{MG}{Lc}\right)^2$, and the shift per revolution becomes

$$\Delta\varphi_{\text{scalar}} = -\pi \left(\frac{MG}{Lc}\right)^2 = -\frac{\pi MG}{c^2 a (1 - e^2)} = -\frac{1}{6} \Delta\varphi, \quad (18)$$

a perihelion *lag* of $7.16''$ per century instead of an advance of $42.98''$, together with no light deflection at all. The reason EINSTEIN required a tensor field is precisely that the spatial curvature is not optional.

Consequence for the present model

Since Eq. (13) is EINSTEIN's structure exactly, the static, spherically symmetric limit of this model coincides with the SCHWARZSCHILD solution, and Eqs. (16) and (17) apply unchanged. The classical solar-system tests — perihelion advance, light deflection and Shapiro delay [8] — are therefore reproduced and are not re-derived here. The model departs from general relativity only in the cosmological regime treated below, where the mass factor q enters and the field becomes self-induced.

3 Energy Conservation and the Equality of the Time Dilations

This section is static. It treats a body in a static field and shows where the equality of the kinetic and the gravitational time dilation, Eq. (1), comes from in that case. It is the equality to which Sect. 2.5 refers.

Setting

A body of mass m falls in the static field of a mass M . Let $f(r) \leq 1$ be the rate of a clock at rest at radius r relative to a clock at rest at infinity, with $f \rightarrow 1$ for $r \rightarrow \infty$. Let v be the velocity of the body as measured by an observer at rest at r , with his

own rulers and clocks. There spacetime is locally flat, so special relativity applies, and $\gamma = 1/\sqrt{1 - v^2/c^2}$.

Derivation

The observer at rest measures the energy γmc^2 . Clocks at r run slow, and energies, like frequencies ($E = h\nu$), are valued smaller by the same factor when seen from infinity, which is the gravitational redshift [9]:

$$E = f(r) \gamma mc^2. \quad (19)$$

Because the field does not change with time, E is conserved along the whole path [10]. A body released from rest at infinity has $f = 1$ and $\gamma = 1$ there, so $E = mc^2$. This is the condition of zero total energy in the Newtonian sense: kinetic and potential energy cancel, as the check below shows. Equating the two expressions for E gives $f\gamma = 1$, that is,

$$\sqrt{1 - \frac{v^2}{c^2}} = f(r). \quad (20)$$

The left side is the kinetic time dilation and contains v ; the right side is the gravitational one and contains only r and M . Their equality is therefore not an additional assumption but energy conservation for a fall from infinity.

The two forms of f

Squaring Eq. (20) gives $v^2/c^2 = 1 - f^2$. With $u \equiv GM/(rc^2)$, the Schwarzschild factor $f = \sqrt{g_{00}} = \sqrt{1 - 2u}$ gives $v^2/c^2 = 2u$, that is the escape velocity v_e of Sect. 2. The first-order factor $f = 1 + \phi/c^2 = 1 - u$ gives $v^2/c^2 = 2u - u^2$, that is Eqs. (1) and (2). The two differ by u^2 , as in Eq. (15). For $v \ll c$ both reduce to $1 - \frac{v^2}{2c^2} \approx 1 - u$, hence $\frac{1}{2}v^2 - \frac{GM}{r} = 0$: the Newtonian total energy is zero.

With the mass factor Q in place, $u = 0.8442$ at $a = 1$ (Sect. 4.2.2). The first-order factor then gives $2u - u^2 = 0.9757$ and $v \approx 0.988 c$, which is the value v_0 of Table 1. The Schwarzschild factor would give $2u = 1.688 > 1$ and no real velocity, the limit already noted after Eq. (29).

What this establishes

The derivation holds for a static, spherically symmetric field and a body released from rest at infinity. Applying Eq. (20) to the time-dependent, self-induced state of the universe is the postulate of the model, not a consequence of this section: conservation along the sequence of cosmic states is not derived, because no equation of motion is available (Sects. 8 and 7.1). Which form of f applies in strong fields is the seam named in Sect. 6.2.

4 Transition to Cosmology

This is the section in which the two readings meet, and both are used in it. While in the Foundations section there was still a distinction between the mass center M

and the mass of the considered object m , cosmological considerations show no such distinction. The gravity of the universe is self-induced, i.e., the sum of the celestial bodies causes its own gravity and moves within it. In this respect, $m = M$.

4.1 The Enclosed Sphere and the Cosmic Horizon

The argument of this subsection is static throughout: it rests on the mass enclosed by a sphere, not on a motion. Everything that follows treats the universe as a homogeneous sphere of radius R and asks after the free fall at its surface. Two objections stand in the way of that step, and both must be answered before the algorithm is set up, because the whole construction rests on them: why may the mass beyond R be left out of account, and does the sphere not place the observer at a distinguished centre?

Why the exterior does not enter

The first objection is answered by a theorem. BIRKHOFF's result [11], obtained earlier by JEBSEN, states that every spherically symmetric solution of the vacuum field equations is static and coincides with SCHWARZSCHILD's. Its corollary is the part that matters here: within a spherically symmetric distribution of matter, the metric interior to a radius R is fixed by the enclosed mass alone,

$$M(R) = 4\pi \int_0^R \rho(r) r^2 dr, \quad (21)$$

and is entirely independent of how the matter outside R is distributed, or of how much of it there is. The point deserves emphasis because it is stronger than it first appears. The exterior contribution is not small, and it does not arrive late; it *cancels exactly*, each parcel of mass-energy being compensated by its counterpart on the opposite side of the sphere. Nothing needs to be shielded, and no appeal to the finite propagation speed of gravitation is required — an appeal that would in any case fail for a universe of unbounded age, in which any such influence would long since have arrived.

This corollary is what has always licensed the Newtonian treatment of cosmology. The cosmological equations for pressureless matter were obtained by NEWTONian means by MCCREA and MILNE [12], and they agree with the relativistic ones precisely because the exterior drops out. The present model stands in that tradition: the sphere of radius R is not an assumption about the extent of the universe, but a statement about which mass governs its interior.

The gravitational horizon

The sphere carries a distinguished radius. Let R_h be the radius at which the enclosed mass closes its own SCHWARZSCHILD condition,

$$\frac{2 M(R_h) G}{R_h c^2} = 1. \quad (22)$$

For a homogeneous sphere of critical density, $M(R) = \frac{4}{3}\pi R^3 \rho_c$ with $\rho_c = \frac{3H^2}{8\pi G}$, the left-hand side becomes

$$\frac{2G}{Rc^2} \cdot \frac{4}{3}\pi R^3 \rho_c = \frac{8\pi G \rho_c}{3} \frac{R^2}{c^2} = \frac{H^2 R^2}{c^2}, \quad (23)$$

so that Eq. (22) is met at

$$R_h = \frac{c}{H}. \quad (24)$$

The HUBBLE radius is therefore not merely a kinematic marker: it is the radius at which the mass it encloses satisfies its own SCHWARZSCHILD condition. MELIA [13] identifies R_h on exactly these grounds — by way of BIRKHOFF’s theorem and its corollary — as a gravitational, or curvature, horizon: the limit beyond which the curvature of the cosmic spacetime prevents any signal from ever having reached the observer. We adopt that identification and nothing further; the $R_h = ct$ cosmology subsequently developed from it is not used here and plays no part in what follows.

The horizon shows itself in the time dilation. With $M(r) = \frac{4}{3}\pi r^3 \rho_c$ the factor of Eq. (13) becomes

$$\tau(r) = \sqrt{1 - \frac{2M(r)G}{rc^2}} = \sqrt{1 - \left(\frac{r}{R_h}\right)^2}, \quad (25)$$

which falls monotonically from unity at the centre to zero at $r = R_h$. A clock at the horizon stands still with respect to one at the centre, and the associated redshift $z = 1/\tau - 1$ diverges there. The interior is thus separated from the exterior not by a screen but by the vanishing of the rate of time — which is the same statement as Eq. (22), read off the metric instead of the mass.

Why this is not a model with a centre

The second objection is answered by homogeneity. The sphere is not a physical object with an edge, and its centre is not a place. Every observer may carry out the construction just given about their own position, and because the matter distribution is homogeneous and isotropic each of them obtains the same R_h and the same $\tau(r)$. No position is distinguished by anything the model contains. What Eq. (21) delimits is the observer’s own horizon, and the universe as a whole may be arbitrarily larger than it — indeed unbounded — without a single quantity computed below being affected. The same homogeneity carries the inference in the opposite direction as well: the interior is not only undisturbed by what lies outside it but representative of it, so that what is established inside the horizon applies beyond it. Sect. 6.2 returns to this point and to the price of the assumption.

This distinguishes the present construction from the static spherically symmetric cosmologies in which the observer is genuinely near a preferred centre, as in ELLIS [14], where the redshift is likewise gravitational rather than kinematic. The observational bounds that constrain such models — the limits on radial inhomogeneity on the scale of the horizon drawn from the thermal spectrum of the CMB [15], and the model-independent test of the COPERNICAN assumption [16] — bear on a preferred position, and the present model asserts none. A related use of gravitational time dilation as the operative effect, rather than as a correction to an otherwise kinematic account, is WILTSHIRE’s treatment of the averaging problem [17].

Consequence for what follows

Every quantity computed in Sects. 5 and 6 refers to the interior of the observer's horizon. Two radii must be kept apart in what follows, and they do not coincide: the cosmic radius R_0 obtained from the algorithm, and the redshift horizon $R_h = c/H_{\text{CMB}}$ of Eq. (24). Their ratio is the HUBBLE tension itself, and the matter is taken up in Sect. 6.2.

4.2 Dark Matter

Conventionally, dark matter is distinguished from baryonic matter. Nevertheless, it is described by a factor relative to it: $\frac{\Omega_m - \Omega_b}{\Omega_b} \approx 5$. We, however, assume that dark matter is directly related to baryonic matter: $q = \frac{\Omega_m}{\Omega_b}$. The idea is to obtain the mass factor q from the relativistic mass (SR):

$$q = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (26)$$

Since assuming a velocity of celestial bodies requires a stationary aether, a purely kinetic consideration is ruled out here. But RICHARD FEYNMAN [1] has advice here too: he treats gravitation as a field in its own right and develops it field-dynamically. With this artifice, one can speak of a velocity v in the universe. A kinematic reading of the expansion is not without precedent: MILNE [18] had already described it as the motion of bodies in flat space rather than as an expansion of space, obtaining the linear velocity–distance relation from the fact that the fastest bodies reach the greatest distances. It is no coincidence that the parameter for velocity is called “ v ” in accordance with the conventions of this article — later we shall see that the celestial bodies are in free fall, even though they are not falling toward a common center.

4.2.1 Mass Factor and Potential Energy

To equip the energy E derived from the speed of light in the gravitational field with the mass factor q , one must distinguish what belongs to the initial state (the Big Bang) from what belongs to the final state under consideration. The initial state is at rest, so q cannot apply to it: q belongs to the motion v .

$$E_{\text{pot}} = M \frac{1}{2} \left(qc^2 \left(1 - \frac{MqG}{Rc^2} \right)^2 - c^2 \right) \quad (27)$$

4.2.2 The Gravitational Form of the Mass Factor

Equation (26) writes q kinetically, through a velocity. Under the second of the two readings announced in the Introduction the same factor can be written without any motion at all, and it is worth doing so once, because in that form the mass factor is a property of the potential. Let

$$u = \frac{MQG}{R_{\text{in}}c^2}, \quad \Phi_Q = -\frac{MQG}{R_{\text{in}}} = -u c^2 \quad (28)$$

be the depth of the potential, with the mass factor Q introduced below in Eq. (37). The free-fall velocity entering Eq. (26) then obeys $v^2/c^2 = 2u - u^2$, so that the radicand is a complete square, $1 - v^2/c^2 = (1 - u)^2$, and the square root drops out:

$$q = \frac{1}{1 - u} = \frac{1}{1 + \frac{\Phi_Q}{c^2}} \quad (29)$$

What makes the square complete is the perturbation term $-u^2$, the very term that is discarded in the weak field. With $u = 0.8442$ at $a = 1$, Eq. (29) returns the same $q_0 = 6.4205$ as the kinetic form, and it does so over the whole expansion history: between $a = 0.02$ and $a = 5$ the two forms differ by at most $1.2 \cdot 10^{-30}$ in relative terms, which is the rounding of the last digit carried by the computation.

That such a factor exists is not a new effect. A body of rest mass m held at rest in a field carries the energy $mc^2\sqrt{g_{00}}$ for the distant observer and the full mc^2 for the local one, so the two bookkeepings differ by $1/\sqrt{g_{00}}$. In the language of mass definitions this is the state of affairs that led TOLMAN [19] to include the contribution of the field in the mass formula, and that later received its own definition as the KOMAR mass [20]. What is new here is not the factor but its identification with dark matter, and that identification is stated as an assumption in Sect. 6 rather than derived.

One limit belongs with Eq. (29) and must be quoted alongside it. With the factor Q the depth reaches $2u = 1.6885 > 1$, so the enclosed mass exceeds its own SCHWARZSCHILD bound and the radicand of the exact expression $\sqrt{1 - 2MQG/(Rc^2)}$ is -0.6885 . That form has no real branch here, whereas FEYNMAN's $1 - MQG/(Rc^2)$ stays real and positive. In this model the latter is therefore not an approximation that could be replaced by the exact form, but the condition under which the calculation exists at all.

4.2.3 Gravitational Acceleration and Force

The potential energy E_{pot} supplemented by the mass factor q yields a gravitational acceleration whose form is at first sight surprising:

$$g_q = \frac{d}{dR} \frac{1}{2} \left(qc^2 \left(1 - \frac{MqG}{Rc^2} \right)^2 - c^2 \right) = q^2 \frac{MG}{R^2} - q^3 \frac{M^2G^2}{R^3c^2} \quad (30)$$

Expressed as the force F in the self-induced universe, however, it becomes intelligible:

$$F = Mg = M \left(q^2 \frac{MG}{R^2} - q^3 \frac{M^2G^2}{R^3c^2} \right) = q^2 \frac{M^2G}{R^2} - q^3 \frac{M^3G^2}{R^3c^2} \quad (31)$$

The powers of M and q are identical, so that the principle of relativistic mass is satisfied.

4.3 Energy Balance of Free Fall

The weightlessness of free fall suggests that the sum of the energies of an object cannot differ from zero. The idea that the universe as a whole carries zero total energy, the positive rest energy of matter being cancelled by the negative gravitational potential

energy, goes back to TRYON [21]; what is new here is that the cancellation is not postulated but follows from the free-fall condition and closes the algorithm. In addition to the energies already mentioned (kinetic energy, energy of relativistic mass, potential energy E), dark energy also carries considerable weight for the universe. A factor x relative to baryonic matter is assigned to it, analogously to the mass factor q :

$$x = \frac{\Omega_\Lambda}{\Omega_b} = \frac{1}{\Omega_b} - q \quad (32)$$

The proven flatness of the universe gives the freedom to replace x in this way, where $\frac{1}{\Omega_b} = \frac{\rho_c}{\rho_b}$. Against this background, the main equation can be established:

$$0 = Mc^2(q - 1) + Mc^2q + \frac{Mc^2}{2} \left(q \left(1 - \frac{MqG}{Rc^2} \right)^2 - 1 \right) - Mc^2x \quad (33)$$

Dark energy ($\Omega_b \cdot x$) counts toward the total mass, but acts repulsively contrary to the other components, and therefore receives a negative contribution. The negativity of the potential energy is left to the internal processes of the derived expression (more on this later). This opens up the possibility of negative gravity, which thus remains not only a component of dark energy.

The bracket in the first term is recognizable when one recalls q (Eq. (26)) and the kinetic energy of SR, and compares it with the relativistic energy: $E_{kin} = Mc^2 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)$ and $M = M_0 \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$.

5 Cosmological Algorithm

The algorithm is kinetic: it follows the trajectory along a , and the quantity it solves for is a velocity. The main equation is a function with many unknowns and variables:

$$f(q, R, M, v) = 0 = Mc^2(2q - 1) + \frac{Mc^2}{2} \left(q \left(1 - \frac{MqG}{Rc^2} \right)^2 - 1 \right) - Mc^2 \left(\frac{1}{\Omega_b} - q \right) \quad (34)$$

Several of these can be eliminated. In a flat universe, $x = \frac{1}{\Omega_b} - q$. Writing $Y = \frac{4}{3}\pi\rho_b$, the total baryonic mass becomes $M = YR_0^3$, an arrangement whose advantage is that M may vary during the iteration. The free-fall velocity v follows from $q = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ as

$v = c\sqrt{1 - \frac{1}{q^2}}$, its original form having been established in Eq. (2).

It thus becomes clear that the mass factor q and the cosmic radius R_0 are entangled and cannot replace each other. To find a way out of this dilemma, there are various possibilities: one can approach the cosmic radius via $R = vt$, but then one

also estimates the age of the universe as an additional unknown. One can also make a statement using the Hubble constant $R_0 = \frac{c}{H_0}$.

The accuracy of the measurement of H_0 using the Planck satellite ($H_0 = 67.4 \pm 0.5 \frac{km}{s \cdot Mpc}$) introduced the concept of precision cosmology. Just a few years later, a measurement using the cosmic distance ladder was to dampen this enthusiasm: the measured value deviated by more than one order of magnitude of Planck's measurement uncertainty. This discrepancy is now known as the Hubble tension, and we return to it in Sect. 6.

None of these estimates of R_0 proved satisfactory. The remaining viable path is to determine the present-day mass factor q_0 from the measured value of $\Omega_{m,0}$: $q_0 = \frac{\Omega_{m,0}}{\Omega_b}$, where $\Omega_{m,0} = 0.315$. Substituting the total baryonic mass (computable from present-day values) $M_0 = YR^3$ into the main equation enables a solution for the present cosmic radius R_0 . The equation¹ is found in App. 9.2.

Solving the cosmological algorithm consistently requires a spatially curved universe. The universe is flat at exactly one instant, and that instant is not $a = 1$, once the measured value $\Omega_k = 0.0007 \pm 0.0019$ [22] is taken at face value.

However, accounting for the curvature factor $k_0 = \frac{\Omega_k}{\Omega_b}$ leads to the following conflict: previously, at $a = 1$, the relation $q + x = \frac{1}{\Omega_b}$ held true. Now, however, relation B) $q + x + k = \frac{1}{\Omega_b}$ applies instead. Since $(q + x) \neq (q + x + k)$, a compensation must be created via Ω_{bs} . This is given by A) $q + x = \frac{1}{\Omega_{bs}}$.

Substituting A into B then yields:

$$\Omega_{bs} = \frac{\Omega_b}{1 - k_0 \Omega_b} \quad (35)$$

With this substitution the algorithm reproduces $k(1) = k_0$ to the working precision of the computation. The equations required to close the algorithm are given in App. 9.2 and 9.3.

With this q_0 , however, the resulting R_0 deviates further than a determination $R_0 = \frac{c}{H_0}$ permits, even allowing for the uncertainty of the Hubble constant and for the Hubble tension.

The expression for R_0 is extremely sensitive to changes in $\Omega_{m,0}$, which prompted a closer examination of the free-fall velocity, since it enters the determination of q_0 and subsequently of $q(a)$:

$$q = \frac{1}{\sqrt{1 - \frac{2 \frac{MG}{R} - \frac{M^2 G^2}{R^2 c^2}}{c^2}}} \quad (36)$$

Evaluated directly, this expression produces unacceptable values for q_0 . Since the main equation depends on the mass factors q and x , it is natural to suppose that the mass entering the free-fall velocity v carries a mass factor as well. We find empirically — a result for which a physical derivation is still outstanding — that the required

¹The result contains seven solutions. To identify the correct one, it is necessary to examine them with suitable parameters. During research, transformations of equations frequently yielded multiple solutions. We ensured that the sought solution is real, positive, nonzero, and plausible.

factor, denoted Q by its dimension, coincides with the corresponding factor for relativistic and kinetic energies in the main equation (34), augmented by the mass factor of dark energy. For a flat universe this gives:

$$Q = 2(q + x) - 1 = \frac{2}{\Omega_{bs}} - 1 \quad (37)$$

The entanglement of q_0 and R_0 can now be resolved by Newton iteration. Since Newton's method drives the iterated function to zero by construction, a second and independent function is required; we use the inner radius $R_{0,in}$ of Eq. (26), see App. 9.1.

The iteration $R_{0,in} - R_{0,out} = 0$, with $R_{0,in}$ from App. 9.1 and $R_{0,out}$ from App. 9.2, is carried out with respect to q and returns q_0 . Using $R_{0,in}(q)$ or $R_{0,out}(q)$ with this q_0 gives R_0 . The matter density parameter that follows, $\Omega_{m,0} = q_0 \Omega_b$, exceeds the measured value by approximately one percent. The total baryonic mass is then $M = Y R^3$, and the mass factor of dark energy $x_0 = \frac{1}{\Omega_{bs}} - q_0$ (see App. 9.3).

5.1 Equivalence to Free Fall

According to the mass factor Q , the free-fall velocity of the universe will be close to the speed of light c . Numerical values become useful here only when they permit a comparison of the two different expressions for v . These are already numerically equivalent in the present via q_0 ; the comparison over the expansion history is carried out in Sect. 6 and shown in Fig. 4.

For a flat universe with constant density parameters, science has developed the scale factor a for the relationship between time and the respective cosmic radius. It moves in time from the Big Bang to the present with values $0 \dots 1$, establishing a proportionality. Only the conversion to the temporal progression reveals the nonlinearity.

We postulate, for lack of better knowledge and regardless of the anticipated variance of the density parameters Ω_m and Ω_Λ , linearity for $a(t)$. Thus, two expressions for the cosmic radius are again sought, but this time over time.

Necessarily, Q can no longer be $\frac{2}{\Omega_{bs}} - 1$ but must account for $Q = 2(q + x) - 1$ in order to compute with the inner equation for q :

$$q = \frac{1}{\sqrt{1 - \frac{2 \frac{M(2(q+x)-1)G}{R_{in}} - \frac{M^2(2(q+x)-1)^2 G^2}{R_{in}^2 c^2}}{c^2}}} \quad (38)$$

The expression for R_{in} is found in App. 9.4. The main equation holds and now has the following form:

$$0 = M c^2 (2q - 1) + M c^2 \left[\frac{1}{2} \left[q \left(1 - \frac{M q G}{a R_0 c^2} \right)^2 - 1 \right] - M c^2 \left(\frac{1}{\Omega_{bs}} - q \right) \right] \quad (39)$$

The decisive background is the basic equation $\Omega_m + \Omega_\Lambda + \Omega_k = 1$ as seen before.

From the main equation, a time-dependent mass factor $q(a)$ can now be determined (a representation analogous to App. 9.3 is inappropriate). Since R_0 and a are known, this yields the desired time-dependent values.

As already noted, R_{out} as a result of the main equation reacts very sensitively to changes in Ω_m or $q(a)$. R_{out} is obtained from the above main equation (39), but without a , since this was consumed for $q(a)$. The expression is found in App. 9.5.

The iteration of $x(a)$ is carried out and following of $q(a)$ via the function $R_{in}(a) - R_{out}(a) = 0$. With this, the curvature factor $k(a) = \frac{\Omega_k(a)}{\Omega_b}$ can also be determined via the mentioned but transformed basic equation $\frac{1}{\Omega_b} = q(a) + x(a) + k(a)$.

6 Results

Table 1 collects the present-day values produced by the algorithm, in the order in which they were introduced above. Figures 1, 2 and 4 show the corresponding trajectories over the expansion history.

Both readings announced in the Introduction are present in this section, and every figure in it is to be read twice. A curve such as $q(a)$ or $x(a)$ may be taken kinetically, as the mass factor belonging to a body moving outwards at the free-fall velocity, or statically, as the mass factor belonging to the potential at that radius. The two are not alternatives between which the reader must choose: by Eq. (1) the associated time dilations are equal, so a single curve carries both meanings at once and no figure would change if it were relabelled from the one reading to the other. This is why the captions name the quantity plotted and not the picture behind it.

Table 1 Present-day values obtained from the cosmological algorithm.

Quantity	Symbol	Value
Cosmic radius	R_0	$1.2775 \cdot 10^{26} \text{ m}$
Mass factor of matter	q_0	6.4205
Total baryonic mass	M	$3.696 \cdot 10^{51} \text{ kg}$
Matter density parameter	$\Omega_{m,0}$	0.31846
Mass factor of dark energy	x_0	13.727
Free-fall velocity	v_0	$2.961 \cdot 10^8 \frac{\text{m}}{\text{s}}$
Age of the universe	t_0	13.756 Gyr

Figure 3 is worth setting beside Fig. 2, because the two are qualitatively alike: both rise from small a to a maximum and fall away again towards the present, and the DESI reconstruction places that maximum at $a \approx 0.74$, close to where $x(a)$ has its own. The quantities differ, however, and the difference in scale should not be mistaken for a discrepancy. Figure 3 shows the dark-energy density normalised to its present value, $\rho_{DE}(a)/\rho_{DE}(1)$, a ratio that is unity today by construction and stays within some ten percent of it. Figure 2, by contrast, shows the mass factor $x(a) = \Omega_\Lambda/\Omega_b$, which measures dark energy against the baryonic density and is therefore of order 14. What the comparison establishes is the shape, not the magnitude: a dark-energy component

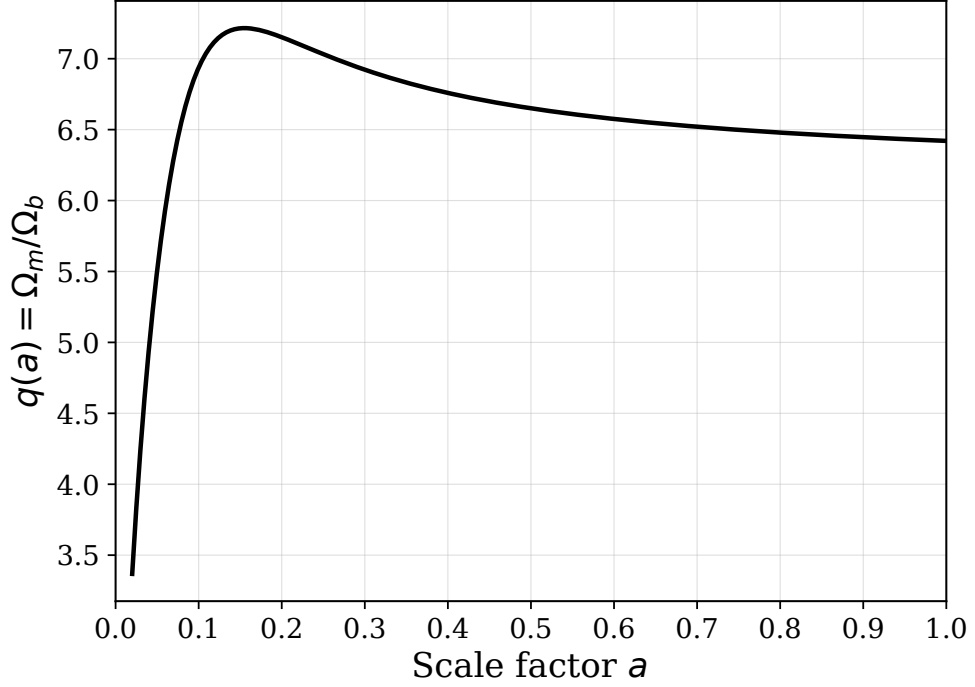


Fig. 1 Mass factor of matter $q(a)$ over $a = 0.02 \dots 1$

that is not constant, with a maximum in the recent past — which is what the DESI data indicate and what the main equation produces here without being asked to.

The two definitions of velocity of free fall are firstly shown in eq. 2 and later by solving $q(a) = \frac{1}{\sqrt{1 - \frac{v_{out}(a)^2}{c^2}}}$ into $v_{out}(a) = c\sqrt{1 - \frac{1}{q(a)^2}}$. The first is written through the potential and is therefore the potential equivalent v_ϕ of Eq. (3); the second is written through the mass factor q and is therefore the velocity itself. It is the same quantity in the two readings, not two quantities.

6.1 Hubble Parameter

6.1.1 Observed Hubble Parameter

The observed Hubble parameter is easy to determine. Its definition is $H_{DL}(t) = \frac{c}{R(t)}$. With the linearity of $a(t)$ postulated in Sect. 5, $a = t/t_0$, so that H_{DL} is computed and plotted as a function of a . The key values can be taken from Fig. 5:

$$H_{DL}(1) = 72.4 \frac{km}{s \cdot Mpc} \text{ and } H_{DL}(0.5) \approx 70 \frac{km}{s \cdot Mpc}$$

The pronounced deviation from the trend predicted by the Λ CDM model makes this parameter the natural test case for the model presented here. An in-depth measurement of H_{DL} , similar to those conducted by Adam G. RIESS and collaborators [27, 28], forms the basis for falsifiability.

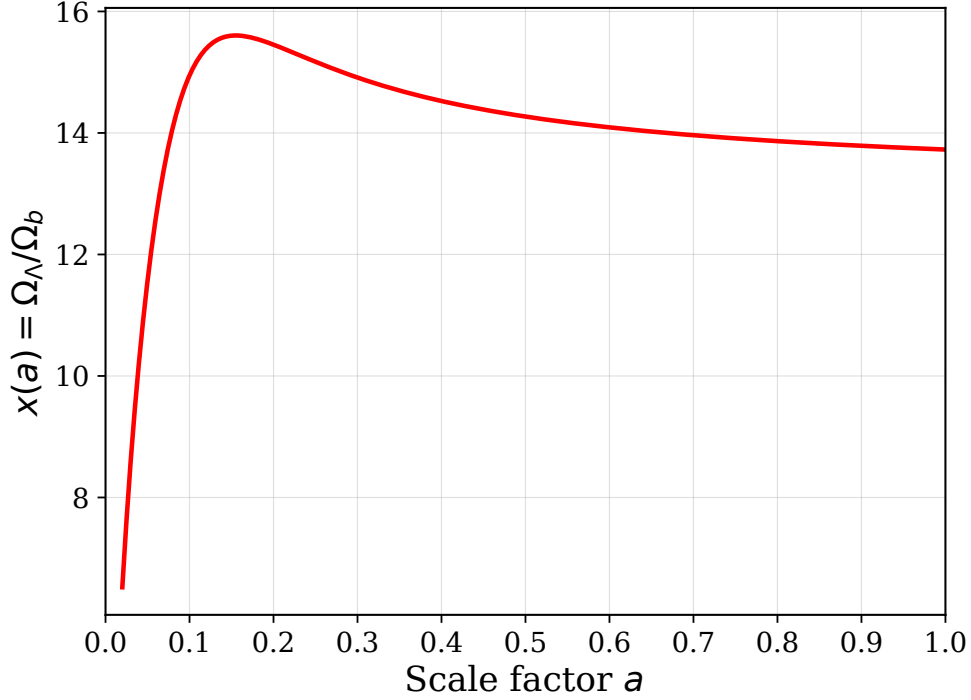


Fig. 2 Mass factor of dark energy $x(a)$ [23] from the main equation (39) over $a = 0.02 \dots 1$, to be compared with Fig. 3

6.1.2 Geometric Hubble Parameter

The geometric Hubble parameter H_{CMB} refers to the Cosmic Microwave Background, the context of its investigation. Its definition is $H_{CMB}(a) = \frac{\dot{a}}{a}$.

That the mass factors of matter $q(a)$ and of dark energy $x(a)$ are no longer constant, and that the universe is not flat, creates a difficulty: no scale factor $a(t)$ exists for time-dependent density parameters in a curved spacetime. We proceed with the conventional form, which is the closest available approximation (Fig. 6):

$$a(t) = \left(\sqrt{\frac{q(t)\Omega_b}{x(t)\Omega_b}} \sinh \left(\frac{3H_0 t \sqrt{x(t)\Omega_b}}{2} \right) \right)^{\frac{2}{3}} \quad (40)$$

Solving Eq. (40) for t gives the cosmic time belonging to a given scale factor,

$$t(a) = \frac{2}{3} \frac{\text{arsinh} \left(a^{3/2} \sqrt{\frac{x(a)}{q(a)}} \right)}{H_0 \sqrt{x(a)\Omega_b}} \quad (41)$$

from which the present age of the universe follows as $t_0 = t(1) = 13.756$ Gyr. The same reservation applies as for Eq. (40) itself: the expression presupposes constant

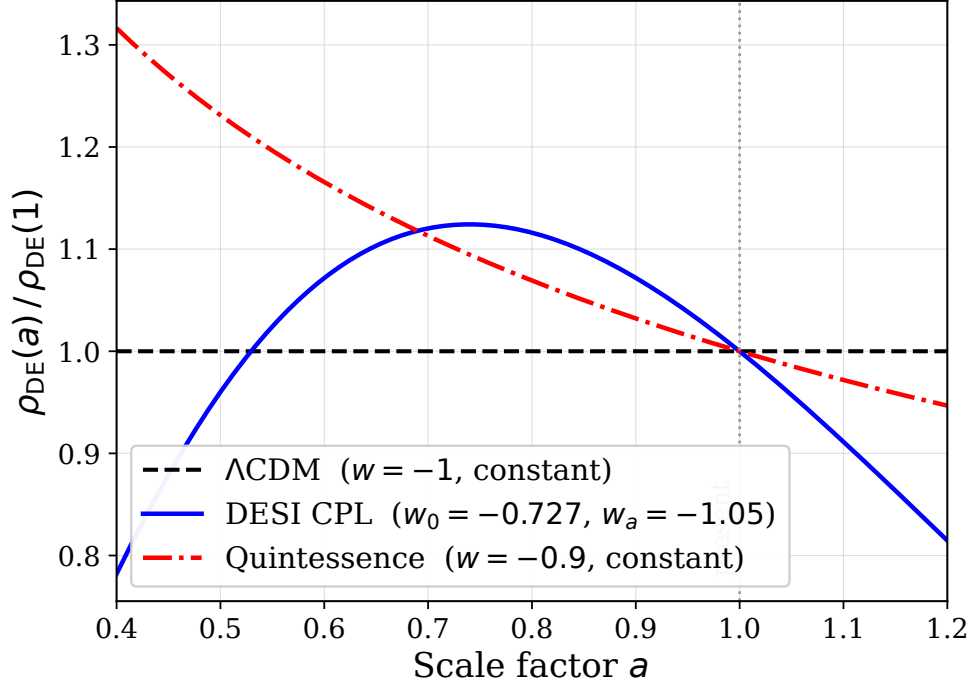


Fig. 3 Evolution of the dark-energy density in the standard picture, for comparison with Fig. 2. Computed from the CPL parametrisation $w(a) = w_0 + w_a(1 - a)$, for which $\rho_{\text{DE}}(a)/\rho_{\text{DE}}(1) = a^{-3(1+w_0+w_a)} \exp(-3w_a(1-a))$, using the DESI DR1 best-fit values $w_0 = -0.727$, $w_a = -1.05$ [24–26]. Shown against Λ CDM ($w = -1$) and a quintessence model with constant $w = -0.9$. By construction all three curves pass through unity at $a = 1$

density parameters and a flat universe, so it is to be read as the best available estimate rather than as an exact result of the algorithm. For comparison, the Hubble time $1/H_0$ amounts to 14.5 Gyr.

As shown in fig. 6 $H_0 \approx 67.4 \frac{\text{km}}{\text{s} \cdot \text{Mpc}}$ in the present and $H(0.5) \approx 125 \frac{\text{km}}{\text{s} \cdot \text{Mpc}}$.

These values may be held against the measurement. The most recent determination of the expansion rate over this range comes from the baryon acoustic oscillations of the DESI second data release, which measure the Hubble rate in seven redshift bins over $0.1 < z < 4.2$ [29]. Fitted with a flat Λ CDM model, those data give $\Omega_m = 0.295 \pm 0.015$ and $H_0 = 68.52 \pm 0.62 \frac{\text{km}}{\text{s} \cdot \text{Mpc}}$, hence $H(a = 0.5) = 120 \frac{\text{km}}{\text{s} \cdot \text{Mpc}}$ with a one-sigma range of $117 \dots 123$. The geometric parameter of the present model lies 4% above that, just outside the interval; at $a = 1$ it is $H_{\text{CMB}}(1) = 67.4 \frac{\text{km}}{\text{s} \cdot \text{Mpc}}$ by construction, since H_0 is an input.

6.2 Redshift and the Two Radii

This subsection is static throughout. The observed redshift has so far been left aside, and it cannot remain so. It is the measured quantity on which the whole of expansion

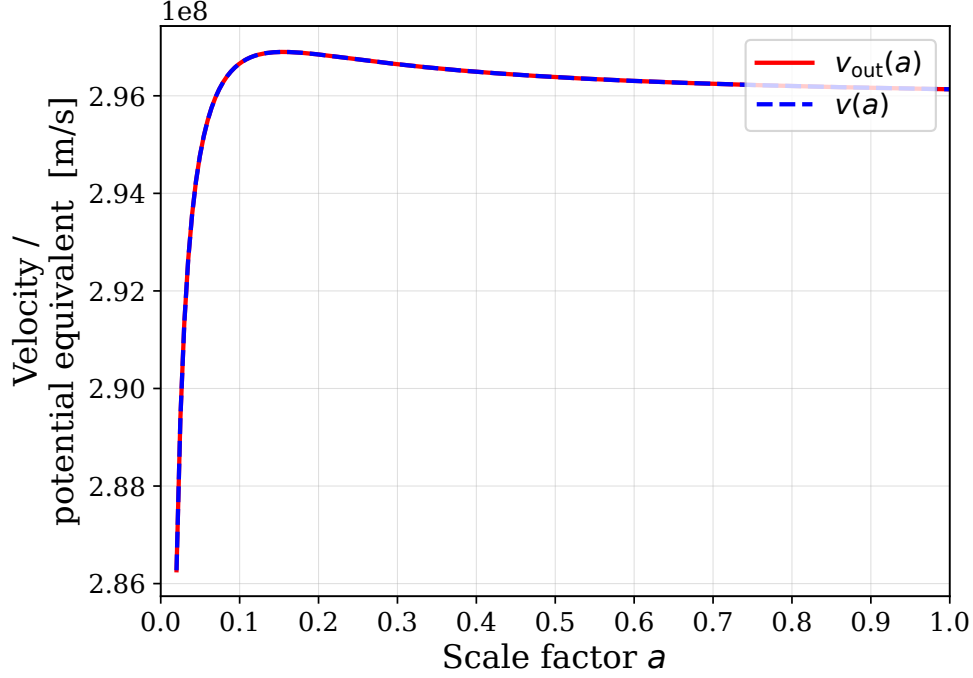


Fig. 4 Free-fall velocity v and test velocity v_{out} over $a = 0.02 \dots 1$. The ordinate carries both readings, as the two time dilations of Eq. (1) are equal: read kinetically it is a velocity, read gravitationally it is the potential equivalent v_ϕ of Eq. (3). The curves are the same under either reading; only the name changes

cosmology rests, and the present model must say what produces it. The question is sharpened by the model's own result: from $a \approx 0.155$ onwards the cosmic radius $R(a)$ *decreases* (Fig. 10) while a continues to grow. A redshift read as recession would have to change sign there. It does not, and the reason is that in this model the redshift is not kinematic at all.

The redshift is gravitational

Section 4.1 has already supplied what is needed. With the time dilation of Eq. (25), a signal emitted at radius r and received at the centre of the observer's horizon arrives shifted by the ratio of the two clock rates,

$$1 + z(r) = \frac{\tau(0)}{\tau(r)} = \frac{1}{\sqrt{1 - \left(\frac{r}{R_h}\right)^2}}, \quad (42)$$

which grows monotonically from zero at the observer and diverges at $r = R_h$. The horizon is therefore not a place where objects become faint; it is the place where they become infinitely redshifted, which is the same statement as Eq. (22). Nothing here

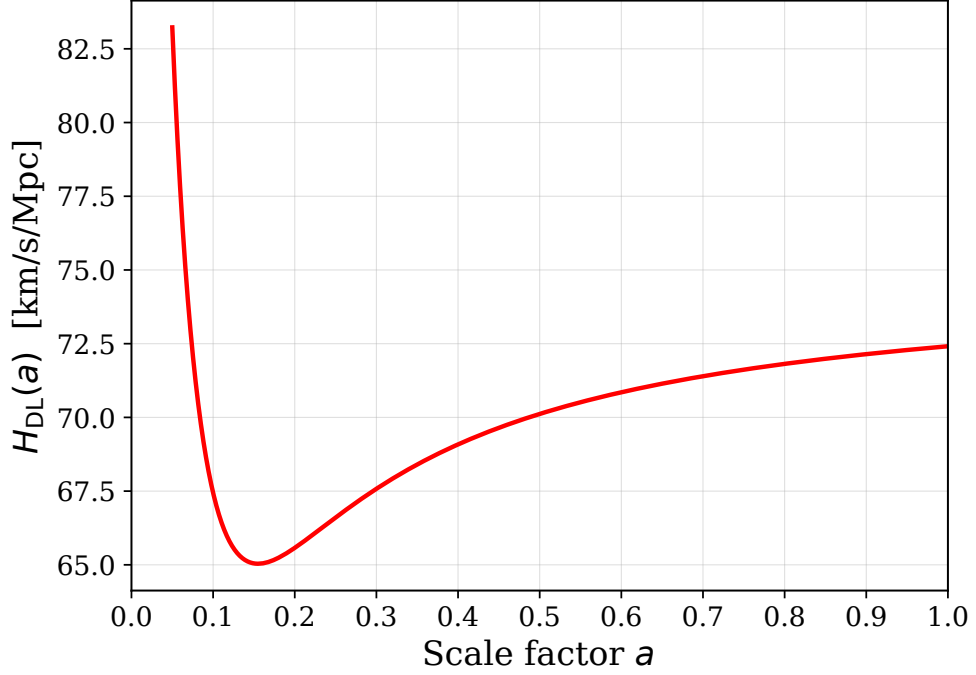


Fig. 5 Observed Hubble parameter $H_{DL}(a)$ over $a = 0.05 \dots 1$, with its minimum of $65 \frac{km}{s \cdot Mpc}$ at $a \approx 0.15$

requires a velocity, and nothing requires space to expand. The interpretation of the cosmological redshift as gravitational rather than kinematic is not new — it is the reading of ELLIS [14] — and what distinguishes the present treatment is set out in Sect. 4.1. Figure 7 shows the relation together with the three radii that matter for it.

Two radii, and their ratio

Eq. (42) refers to $R_h = c/H_{CMB}$, not to the cosmic radius R_0 delivered by the algorithm. The two are different quantities and they take different values,

$$R_0 = \frac{c}{H_{DL}} = 1.2775 \cdot 10^{26} \text{ m}, \quad R_h = \frac{c}{H_{CMB}} = 1.3725 \cdot 10^{26} \text{ m}, \quad (43)$$

whose ratio is

$$\frac{R_h}{R_0} = \frac{H_{DL}}{H_{CMB}} = \frac{72.4}{67.4} = 1.074. \quad (44)$$

The discrepancy between the two radii is thus not an additional assumption but the HUBBLE tension itself, expressed as a length. The observer's redshift horizon lies some seven per cent beyond the cosmic radius, and the two coincide only if the two measured values of the HUBBLE constant coincide.

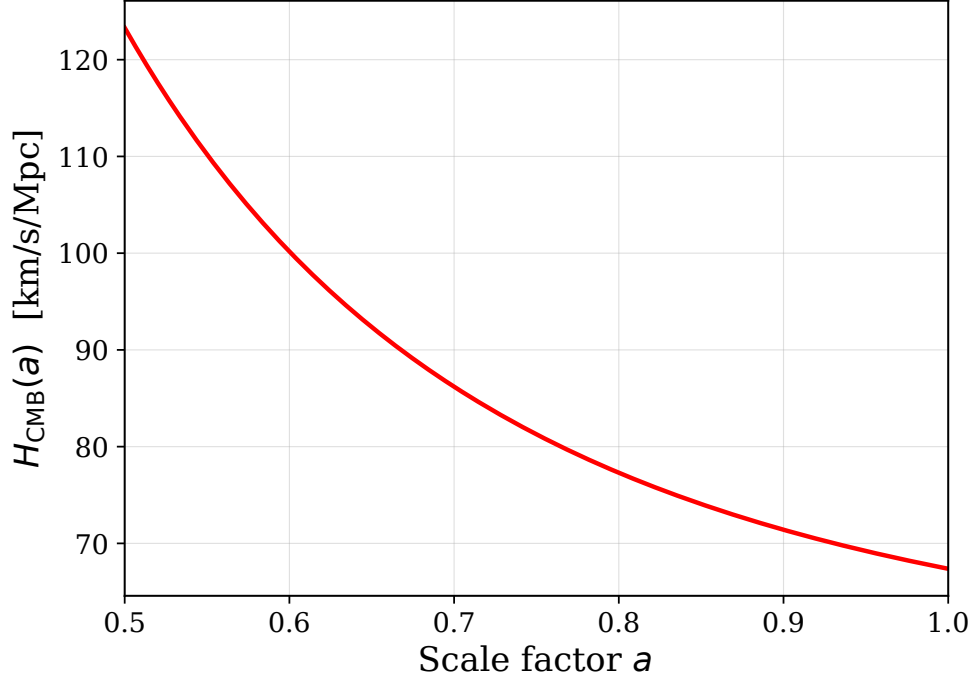


Fig. 6 Geometric Hubble parameter $H_{CMB}(a)$ over $a = 0.5 \dots 1$

A check falls out of this without being arranged. At the radius at which the value v_0 of Sect. 6 is reached, $r/R_h = v_0/c = 0.9878$, Eq. (25) gives $\tau = \sqrt{1 - v_0^2/c^2} = 1/q_0$ and hence

$$z = \frac{1}{\tau} - 1 = q_0 - 1 = 5.4205, \quad (45)$$

so that the gravitational and the kinematic readings agree exactly at that radius. Neither expression was adjusted to bring this about.

Two forms of the time dilation, and why

It must be stated openly that two forms of the gravitational time dilation are in use, and that they are not interchangeable. Inside the algorithm the first-order form $1 - MG/Rc^2$ of Eq. (1) is used, as it must be: with the mass factor Q of Eq. (37) in place, the argument of the exact expression is $1 - 2MQG/R_0c^2 = -0.688$ and the root is not real. On the line of sight, by contrast, the exact form of Eq. (13) is used, and there it must be, because the first-order form saturates at $z = 1$ and cannot reach the redshifts that are observed.

The licence for this is Eq. (15): the two forms agree to first order and separate at second, the first-order form being the leading term of a series that converges to g_{00} . Where the potential is shallow the leading term suffices; on a line of sight reaching

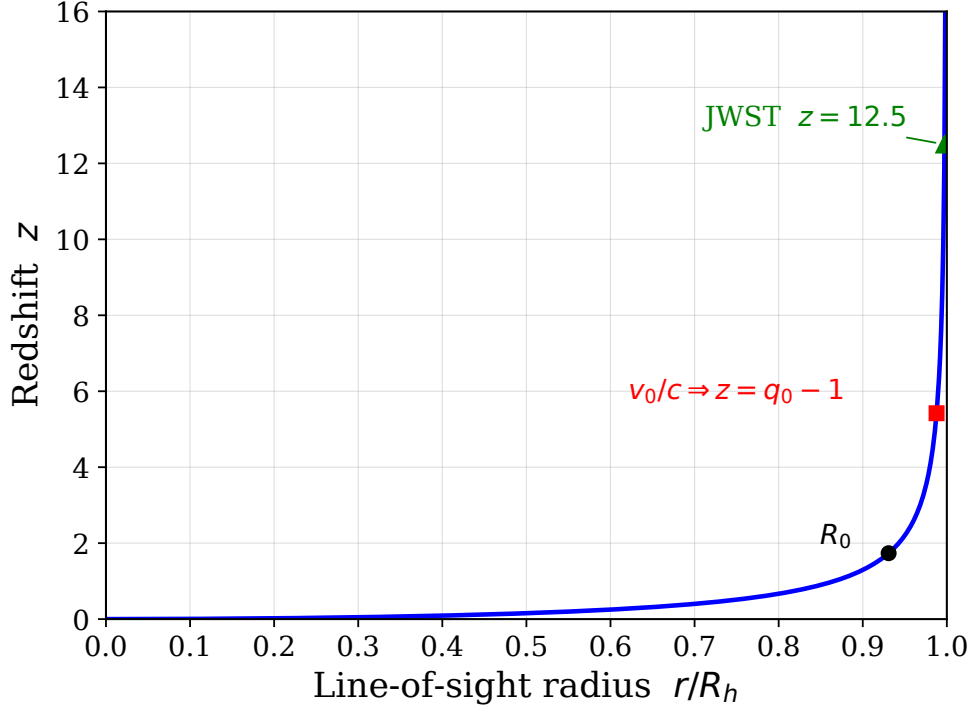


Fig. 7 Redshift z against the line-of-sight radius r in units of the redshift horizon $R_h = c/H_{CMB}$, from Eq. (42). The curve is a static relation: it assigns a redshift to a radius, not to a recession velocity, and the kinetic reading enters only through the marked point $r/R_h = v_0/c$, at which $z = q_0 - 1 = 5.42$ follows without any adjustment. Also marked are the cosmic radius R_0 , which lies at $r/R_h = 0.93$, and the observed galaxies at $z = 12.5$, which lie at 99.7% of R_h and therefore outside R_0

towards the horizon it does not. The distinction is one of regime, not of convenience — but it is a seam in the argument, and it is better named here than found later.

The observed high-redshift galaxies

The galaxies at $z \approx 12.5$ reported from JWST [30, 31] follow from Eq. (42) at

$$\frac{r}{R_h} = \sqrt{1 - \frac{1}{(1+z)^2}} = 0.9973, \quad r = 1.3687 \cdot 10^{26} \text{ m}, \quad (46)$$

that is, at 99.7% of the redshift horizon — and thus outside the cosmic radius R_0 . This is a consequence of Eq. (43) and is stated here rather than avoided: the model does not identify R_0 with the limit of visibility. The limit of visibility is R_h , and the deepest objects yet observed lie close to it.

The early phase, $a = 0 \dots 0.15$

The same distinction settles what the early phase of the model describes. Over that interval the cosmic radius and the redshift horizon are far apart — the horizon being the smaller of the two because $H_{CMB} = \dot{a}/a$ was large in the early phase, rising from 67.4 today to some $2.2 \cdot 10^4 \frac{\text{km}}{\text{s} \cdot \text{Mpc}}$ at $a = 0.01$, while $R(a)$ varies by little more than a factor of two (Table 2 and Fig. 8):

Table 2 Cosmic radius $R(a)$ against the redshift horizon $c/H_{CMB}(a)$ of the same epoch, in 10^9 light years.

a	$R(a)$	$c/H_{CMB}(a)$	ratio
0.02	7.7	0.10	77
0.10	14.5	0.78	19
0.155	15.0	1.47	10
0.50	13.9	7.9	1.8
1.00	13.5	14.5	0.93

In the early phase the sphere treated by the algorithm was ten to seventy times larger than the region an observer inside it could see. The horizon grew into the sphere and overtook it only recently, at $a = 0.886$. What the limit $a \rightarrow 0$ describes is accordingly the state of the matter enclosed by one horizon, not the origin of everything: by Eq. (21) that interior never depended on what lay outside it, and by the homogeneity invoked in Sect. 4.1 every other observer’s sphere passes through the same states at the same time. The Big Bang of this model happens everywhere, has no point of origin, and is compatible with a universe arbitrarily larger than R_h .

Two properties of that interior hold at once, and together they carry further than either of them alone. By Eq. (21) the sphere is undisturbed: what lies outside it does not enter, because it cancels exactly. By the homogeneity invoked in Sect. 4.1 the sphere is at the same time representative: it occupies no distinguished position, and every other observer’s sphere passes through the same states. What is obtained here about density, curvature and expansion history is therefore not confined to the region that can be seen; it holds equally for the part of the universe beyond the horizon, which no observation will ever reach. This follows under the assumed homogeneity, which is a premise of the model and not a result of it: homogeneity beyond the horizon cannot be established observationally even in principle, because observations are made on the past light cone and not on spatial surfaces [32].

What the sphere is, and how far back it reaches

Three questions follow immediately, and it is better to answer them here than to leave them to the reader.

The first is which of the two radii carries the mass. It is the sphere, not the horizon. The total baryonic mass $M = YR_0^3$ is a constant of the algorithm; the radius that

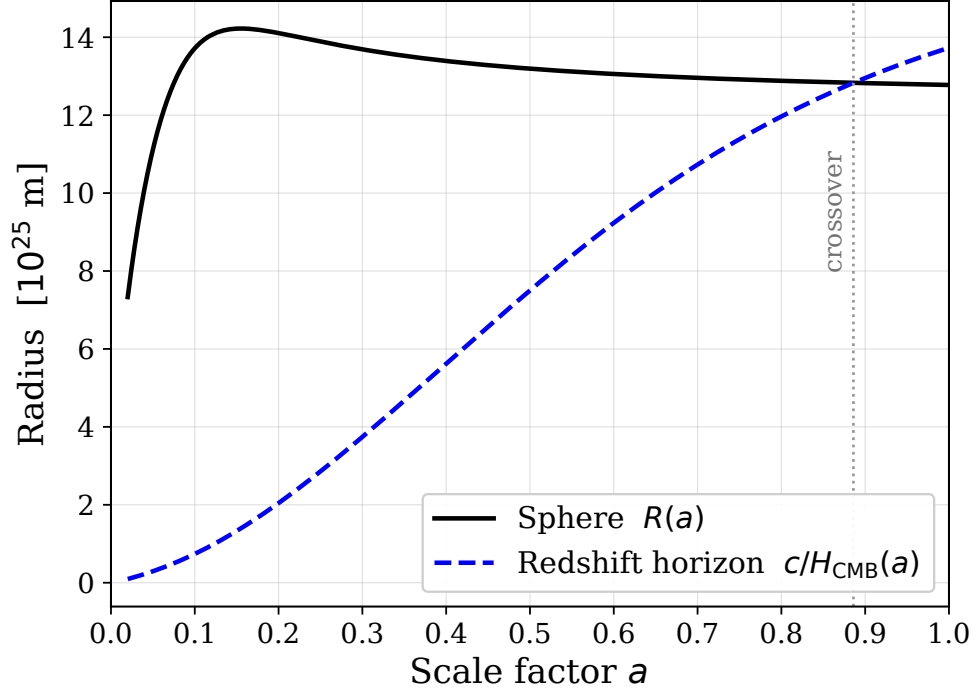


Fig. 8 The two radii of Table 2 over $a = 0.02 \dots 1$: the sphere $R(a)$ treated by the algorithm, which carries the constant baryonic mass M and passes through its maximum at $a \approx 0.155$, against the redshift horizon $c/H_{CMB}(a)$ of the same epoch. Both readings give the same two curves — the sphere is where the free fall takes place and equally where the enclosed mass sets the potential. The horizon grows into the sphere and overtakes it at $a = 0.886$; before that, most of the sphere was never visible from inside it

encloses it is $R(a)$, and the redshift horizon $c/H_{CMB}(a)$ encloses it only from $a = 0.886$ onwards. Reading Table 2 the other way round — as though the visible region had always contained M — is precisely the equivocation in a warned against below, and it leads to a mean density that the model does not have.

The second concerns that density. Since M is fixed and $R(a)$ is not, the mean baryon density inside the sphere is $\rho(a) = \rho_b (R_0/R(a))^3$. It is therefore governed by the radius and not by the scale factor: it falls from $5.29 \rho_b$ at $a = 0.02$ to a minimum of $0.72 \rho_b$ at the radius maximum $a \approx 0.155$ and rises again to ρ_b today — a variation by a factor of about seven, where a^{-3} would demand 10^5 . This is not a contradiction of the standard picture but a statement about two different volumes: a^{-3} refers to a comoving volume, whereas $R(a)$ is not proportional to a and passes through a maximum. Inside the sphere the mean density accordingly falls until that maximum and rises slightly afterwards, by a factor of about seven in all, and not monotonically.

The third is how far back the trajectory reaches. Not to $a = 0$. Following the branch downwards, $q(a)$ and $x(a)$ both decrease towards unity and the solution ends at $a = 0.00302$, where $q = 1$ and hence $v_{out} = c\sqrt{1 - 1/q^2} = 0$: the potential equivalent

vanishes together with the potential itself, the root for x merges with the pole of R_{out} , and $R(a)$ grows without bound. Below that point $q < 1$ and it would be imaginary. The model therefore does not describe a singularity and does not reach one; it describes a state of finite density and vanishing potential equivalent, and stops there. Whether anything preceded that state is a question this model cannot answer, because it fixes $R(a)$ algebraically and supplies no equation of motion to integrate backwards through it.

What remains open

Two reservations belong here. The scale factor a is used in Eq. (40) in its conventional sense and elsewhere as a linear parameter of time; the two radii of Eq. (43) keep the corresponding lengths apart, but a derivation of $a(t)$ for time-dependent density parameters in a curved spacetime is still outstanding, as Sect. 6 has already noted. And Eq. (42) is a static relation: it gives the redshift belonging to a radius, not to a look-back time. A treatment of the propagation of light through the time-dependent field is the natural next step and is not attempted here.

7 Consequences

What follows is stated in the static reading, as befits a conclusion about an end state. The idea of a spatially curved universe did not arise only with the result that the expansion proceeds at the potential equivalent v_ϕ . The measured value of $\Omega_k = 0.0007 \pm 0.0019$ [22] had already attracted our attention. The value itself is an input to the algorithm and is not recovered by it. What the algorithm supplies is the curvature factor $k(a)$ as a function of the expansion history, and the fact that the results depend on the input only weakly: varying Ω_k over the full Planck interval $-0.0012 \dots 0.0026$ changes q_0 and $\Omega_{m,0}$ by 0.18 %, R_0 and H_{DL} by 0.08 %. The epoch of exact flatness, $k(a) = 0$, lies at $a = 0.982$ and not at $a = 1$. We therefore already observe a nearly static universe at the present epoch (Fig. 10). Over the next 50 to 100 Gyr its size will decrease by less than 3 percent, towards a final value of approximately $1.24 \cdot 10^{26}$ m.

7.1 Stability of the Static End State

A model that puts a static universe in its title inherits a question that is older than the model. EINSTEIN's static world is an unstable equilibrium, as EDDINGTON showed [33], and the objection has to be met rather than passed over.

Why the classical instability does not carry over

The instability is a property of how EINSTEIN's solution is held static. There the acceleration

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho + \frac{3p}{c^2} \right) + \frac{\Lambda c^2}{3} \quad (47)$$

is made to vanish at a chosen value of a by fixing Λ against the matter density, $\Lambda c^2 = 4\pi G \rho_m$ for pressureless matter. Displacing the radius by $a \rightarrow a(1 + \varepsilon)$ dilutes the matter as $\rho_m(1 - 3\varepsilon)$ while Λ stays put, so that

$$\frac{\ddot{a}}{a} = 4\pi G \rho_m \varepsilon \quad \implies \quad \ddot{\varepsilon} = 4\pi G \rho_m \varepsilon, \quad (48)$$

and the displacement grows exponentially. The equilibrium exists only because two independent quantities were tuned against each other, and it fails for the same reason.

Neither element is present here. Nothing in the algorithm is adjusted to make an acceleration vanish, and dark energy does not enter as a constant to be balanced against matter: it enters as the factor $x(a)$ of Eq. (32), which is itself determined by the closure and varies over the expansion history (Fig. 2). The coincidence EINSTEIN had to arrange therefore does not arise, and Eq. (48) has nothing to act upon.

What the trajectory does instead

The static state of this model is not an equilibrium point but the asymptotic value of a trajectory. Continuing the algorithm beyond the present epoch gives
with the limiting value

Table 3 Approach to the static end state. The radius falls monotonically and its rate of change vanishes as a^{-2} .

a	$R(a)/R_0$	dR/da [10^{22} m]
1	1.0000	−426.8
2	0.9832	−107.2
5	0.9732	−17.12
10	0.9698	−4.272
20	0.9682	−1.067
50	0.9672	−0.171

$$\lim_{a \rightarrow \infty} \frac{R(a)}{R_0} \simeq 0.9665, \quad (49)$$

extrapolated from the last two rows. The approach is monotone, without oscillation and without runaway, and the rate of change falls off as a^{-2} . In the variable a the static state is an attractor rather than a balance, and it is reached from one side only.

What this establishes and what it does not

It should be said plainly what has and has not been shown. Table 3 establishes that the trajectory delivered by the algorithm converges to a finite radius, and Eqs. (47) and (48) establish that the mechanism of EDDINGTON’s instability has no counterpart here. That is not the same as a stability analysis in his sense. The present model fixes $R(a)$ algebraically, through the closure, rather than by integrating an equation of motion; there is consequently no state to which a perturbation could be applied and then propagated in time. A linear stability analysis presupposes the dynamical equation whose absence is already acknowledged in Sect. 6.2, and supplying it is the prerequisite for settling the question properly. What can be said at present is that the model exhibits no instability and contains no tuned balance that could produce one.

Furthermore, the mass factors of matter and of dark energy both decrease towards the Big Bang (Figs. 1 and 2). Followed to the end of the branch they approach unity rather than zero, at $a = 0.00302$, where the potential equivalent vanishes and the solution terminates; the plotted range $a \geq 0.02$ shows only the upper part of that descent. What holds throughout, and does not depend on the limit, is that the universe persists with zero total energy. Divided by Mc^2 , the main equation reads

$$(2q(a) - 1) + \frac{1}{2} \left(q(a) \left(1 - \frac{Mq(a)G}{R(a)c^2} \right)^2 - 1 \right) - x(a) = 0 \quad (50)$$

and is satisfied over the whole expansion history to the full working precision of the computation: over $a = 0.01 \dots 1$ the residual of Eq. (50) stays below $3 \cdot 10^{-30}$ in magnitude, so that what remains is the rounding of the thirtieth digit and not a deviation.

It is therefore difficult to maintain that the universe is, and has always been, spatially flat. Looking at the curvature factor $k(a)$, (cf. Fig. 9), it is much more natural to understand the early universe in terms of spatial curvature.

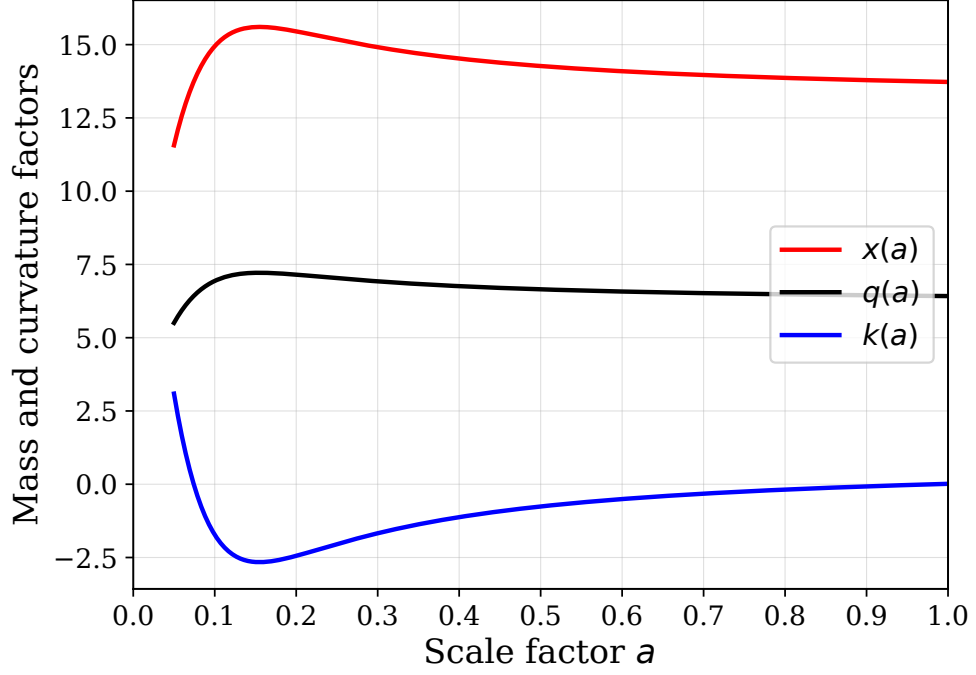


Fig. 9 Mass factors $q(a)$ and $x(a)$, and curvature factor $k(a)$ from the inner equation (38) over $a = 0.05 \dots 1$

A transition from an extremely hyperbolic spacetime into a spherical one is easier to comprehend than the compression of all matter into a point, that is, a singularity. Also in light of fully formed galaxies at the edge of the visible universe at $z = 12.5$ [30] and $z = 12.34$ [31], the early temporal progression of the cosmic radius $R(a)$, shown in Fig. 10, provides an anchor for explaining its formation by the peak in $a \approx 0.2$. Finally, the question of the cosmic fog wall - which appears, though deep in the universe, amid fully developed galaxies - remains unresolved.

The closure relation $\Omega_b (q(a) + x(a) + k(a)) = 1$ is likewise satisfied over the whole range.

That we happen to live at an epoch in which the flatness of the universe must be debated, because the measurements admit both possibilities, is itself worth remarking upon.

Tables 5 and 6 in the Appendix compare this model with the Λ CDM standard model.

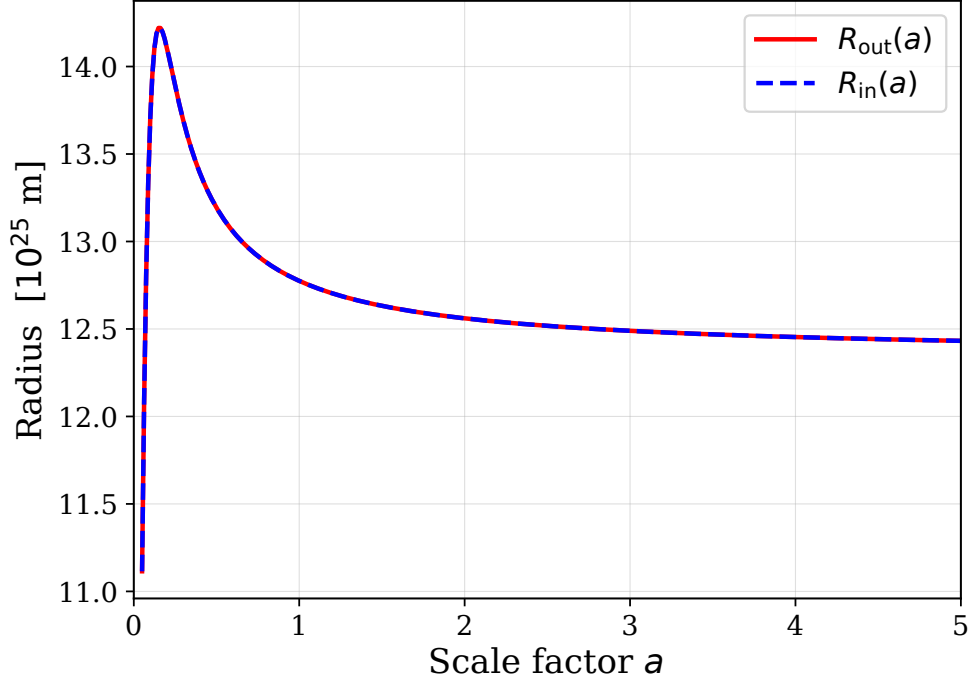


Fig. 10 Cosmic radius $R_{out}(a)$ and $R_{in}(a)$ over $a = 0.05 \dots 5$; the two curves coincide, which is the closure condition of the algorithm. Beyond $a = 1$ the curve is the future: after the maximum at $a \approx 0.15$ the radius falls monotonically towards its static value of about $1.24 \cdot 10^{26}$ m

7.2 What This Model Does Not Treat

It is proper to end by saying which things a cosmological model is normally expected to supply and this one does not. They are named here without apology, because they are not oversights but consequences of what the model is.

It makes no statement about the *spatial distribution* of mass. The mass factor $q = \Omega_m/\Omega_b$ is a global number, and a global number cannot reproduce the observations that established dark matter as a distinct component: the shape of galactic rotation curves, the spatial separation of the lensing mass from the baryons in merging clusters, or the height of the third acoustic peak, which fixes the ratio of collisionless to baryonic matter through physics acting before recombination. The present work does not claim to explain any of these, and q should be read as a parametrisation of a measured ratio rather than as an account of its origin.

It supplies no perturbation theory, and with it no acoustic spectrum, no structure formation and no primordial abundances. It also leaves the age of the universe undetermined: Eq. (40) presupposes constant density parameters and flatness, both of which this model denies, so the value $t_0 = 13.756$ Gyr is the best available estimate rather than an output of the algorithm, and a route internal to the model is not available because $R(a)$ does not vanish as a decreases. Finally, the agreement of the static,

spherically symmetric limit with the SCHWARZSCHILD solution (Sect. 2.5) is an identification and not a derivation; the model determines the time component of the metric and says nothing about the spatial one.

These four have a single common cause. The model fixes $R(a)$ algebraically, through the closure condition, and never integrates an equation of motion. Everything that requires one — the growth of perturbations, a look-back time, a stability analysis in EDDINGTON’s sense — is therefore out of reach, and the mathematical apparatus available for such questions is in any case built around the Λ CDM background, whose assumptions this model does not share. Supplying that equation of motion is the prerequisite for all four, and it is the direction in which this work would have to be continued.

8 Concluding Remarks on the Working Principle

A reader trained on the Λ CDM model looks for an equation of motion: the Friedmann equation, integrated in time, gives $a(t)$ and with it the radius. The present model has no such equation. This section states how the universe works in this model and what is not claimed. Nothing is added to what the preceding sections contain; they are cited rather than repeated.

A family of closed states

Each value of a defines a state. In it the main equation (39) closes the energy to zero, Eq. (50), and the closure relation $\Omega_b(q(a) + x(a) + k(a)) = 1$ of Sect. 5 fixes the mass factors. The radius $R(a)$ is not obtained by integrating a velocity but algebraically, from $R_{in}(a) = R_{out}(a)$ (Eq. (38) and App. 9.5). The sequence of these states behaves like a flip book: every frame is self-consistent, and the law that would drive the system from one frame to the next is not formulated. This is the common cause of the limits listed in Sect. 7.2.

What links the frames

The link is the equality of the time dilations, Eq. (1). It allows the outward velocity v and the potential equivalent v_ϕ of Eq. (3) to be exchanged, and with them the mass factor in its kinetic form, Eq. (26), and in its gravitational form, Eq. (29). A motion can thus be carried from frame to frame as a property of the potential at that radius, without loss. Sect. 3 shows that the equality is energy conservation in a static field; using it in every frame is the postulate.

Why v is not dR/dt

In the Newtonian and the Friedmann picture the velocity at the edge of a sphere is the rate of change of its radius. Here it is not: v is the potential equivalent at the edge, while $R(a)$ is fixed by the closure. At $a = 1$ the velocity is $v_0 = 2.961 \cdot 10^8$ m/s (Table 1), whereas $dR/da = -4.268 \cdot 10^{24}$ m (Table 3). With the linear time parameter $a = t/t_0$ of Sect. 6 and $t_0 = 13.756$ Gyr this is $dR/dt = (dR/da)/t_0 \approx -9.83 \cdot 10^6$ m/s, some thirty times smaller than v_0 and of opposite sign. The magnitude depends on that postulated parameter; the sign does not, since R falls after its maximum at $a \approx 0.155$ (Fig. 10). The two quantities answer different questions: v describes the state of motion, dR/dt how the radius changes from frame to frame.

Why the redshift keeps its sign

A redshift read as recession would change sign where R begins to fall. In this model it is gravitational, Eq. (42), a property of the line of sight to the redshift horizon $R_h = c/H_{\text{CMB}}$ of Eq. (43); the radius R may therefore fall while the redshift persists (Sect. 6.2).

Free fall needs no force

In free fall bodies are weightless: the heavy mass responding to the field equals the inertial mass resisting a change of motion, so no force drives the matter and no inertial resistance has to be overcome. The force that follows from Eqs. (9) and (12) is a locally readable quantity and not a drive, since it is not embedded in an equation $F = m\ddot{r}$. This does not replace the missing equation of motion for $R(a)$: the passage of the radius through its maximum is likewise fixed algebraically.

What is claimed, and what is not

Claimed is that for every a there is a state with zero total energy and closure, and with it the curves $q(a)$, $x(a)$, $k(a)$, $H_{\text{CMB}}(a)$ and $H_{\text{DL}}(a)$ (Figs. 9, 6, 5), the approach to a static end state (Table 3) and, as a test, the minimum of H_{DL} at $a \approx 0.15$. Along the sequence of frames energy conservation is not derived; zero energy holds in each frame as a closure. Not claimed are a causal mechanism for the transition between frames, a stability analysis in EDDINGTON's sense (Sect. 7.1) and perturbation theory (Sect. 7.2). Supplying an equation of motion for $R(a)$ is the next step.

9 Appendix

9.1 Appendix A

Generation of the expression for the inner present cosmic radius:

$$\begin{aligned} \sqrt{1 - \frac{v^2}{c^2}} &= 1 - \frac{Y R^3 \left(\frac{2}{\Omega_{bs}} - 1 \right) G}{R c^2} \implies q = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \\ \implies R_{0,in} &= \frac{-c}{G q Y \Omega_{bs} - 2 G q Y} \sqrt{-G q Y (\Omega_{bs} - 2) \Omega_{bs} (q - 1)} \end{aligned} \quad (51)$$

9.2 Appendix B

Generation of the expression for the outer present cosmic radius:

$$\begin{aligned} R_{0,out} &= \frac{1}{Y q_0 G} \sqrt{\frac{Y q_0^2 G c^2 \Omega_{bs} - \sqrt{D}}{q_0 \Omega_{bs}}}, \\ D &= -6 Y^2 q_0^4 G^2 c^4 \Omega_{bs}^2 + 3 Y^2 q_0^3 G^2 \Omega_{bs}^2 c^4 + 2 Y^2 q_0^3 G^2 \Omega_{bs} c^4 \end{aligned} \quad (52)$$

9.3 Appendix C

Generation of $q(a)$ from the main equation:

$$\begin{aligned} 0 &= M c^2 (q - 1) + M c^2 q \\ &\quad + M c^2 \left[\frac{1}{2} \left[q \left(1 - \frac{M q G}{a R_0 c^2} \right)^2 - 1 \right] \right] \\ &\quad - M c^2 \left(\frac{1}{\Omega_{bs}} - q \right) \end{aligned} \quad (53)$$

A representation of $q(a)$ is inappropriate, as it is a very large and unwieldy expression that must be generated electronically.

9.4 Appendix D

Generation of the expression for R_{in} as the result of the inner equation:

$$\begin{aligned} R_{in} &= \frac{1}{\left[2 \left(c^4 - \frac{1}{q^2} c^4 \right) \right]} \left(4 M G q c^2 + 4 M G x c^2 + 2 M G c^2 \right. \\ &\quad \left. - 2 G c^2 \frac{M}{q} + 4 G c^2 \frac{M}{q} x \right) \end{aligned} \quad (54)$$

9.5 Appendix E

Cosmic radius from the main equation:

$$R_{out} = \frac{2c^2q^2MG + 2\sqrt{E}}{2(5qc^4 - 3c^4 - 2c^4x)}, \quad (55)$$

$$E = -4c^4q^4M^2G^2 + 3q^3M^2G^2c^4 + 2q^3M^2G^2c^4x$$

9.6 Physical Constants and Parameters Used

Table 4 Constants and measured parameters used throughout. The cosmological parameters H_0 , Ω_b and Ω_k are taken from Planck 2018 [22]; the baryonic matter density follows from them as $\rho_b = \rho_c \Omega_b = \frac{3H_0^2}{8\pi G} \Omega_b$.

Quantity	Symbol	Value
Megaparsec	1 Mpc	$3.085677581 \cdot 10^{22}$ m
Gravitational constant	G	$6.67430 \cdot 10^{-11}$ m ³ kg ⁻¹ s ⁻²
Speed of light	c	299 792 458 m s ⁻¹
Hubble constant	H_0	67.4 ± 0.5 km s ⁻¹ Mpc ⁻¹
Baryonic density parameter	Ω_b	0.04960
Spatial curvature parameter	Ω_k	0.0007 ± 0.0019
Baryonic matter density	ρ_b	$4.232 \cdot 10^{-28}$ kg m ⁻³

9.7 Comparison Table

Table 5 Overview of the Λ CDM Standard Model.

Property / Parameter	Characteristics / Value
Fundamental principle	Friedmann equations
Free parameters	Six, fitted to the data [22]
Energy balance of the universe	Not necessarily zero
Hubble tension & Unresolved	67.4 vs. ~ 73 km/s/Mpc
Matter density parameter (Ω_m)	Constant (standardly scaled)
Dark energy (Ω_Λ)	Cosmological constant (Λ)
Spatial curvature (Ω_k)	Exactly 0 (postulated for flat universe)
Scale factor $a(t)$	Standard vacuum energy solution
Big Bang singularity	Mathematical point of infinite density

Table 6 Overview of the Presented Cosmological Model (Zero-Energy / Free-Fall Expansion).

Property / Parameter	Characteristics / Value
Fundamental principle	Free fall / zero-G acceleration
Free parameters	None; three measured inputs (H_0 , Ω_b , Ω_k)
Energy balance of the universe	Exactly zero (self-induced)
Hubble tension & Unresolved	Resolved ($H_{CMB} = 67.4$, $H_{DL} = 72.4$)
Matter density parameter (Ω_m)	Time-dependent / variable ($q(t)\Omega_b$)
Dark energy (Ω_Λ)	Dynamic mass factor $x(t)$
Spatial curvature (Ω_k)	Dynamically determined ($\Omega_k = 0.0007 \pm 0.0019$)
Scale factor $a(t)$	Analytical modification (Eq. (40))
Big Bang singularity	Bursting from hyperbolic spacetime

Declarations

Data Availability Statement

The primary data from the Dark Energy Spectroscopic Instrument (DESI) used in this study are openly available from the official DESI repositories. The reconstructed datasets and trajectories generated during the current study are available from the corresponding author upon reasonable request.

Acknowledgments

The author thanks Jörg Hansmann, who organised and carried through the computational side of this collaboration: he set up and ran the calculations, mediated between the MathCad worksheets and their computational counterpart, and prepared the correspondence on which the revisions of this manuscript are based. The Python implementation is available from the author on request.

Two questions are left open here and are offered to the interested reader. The first is the mass factor $Q = 2(q+x) - 1$ of Eq. (37). It has no latitude: replacing it by $2q - 1$, by $q + x$ or by unity makes the potential equivalent and the velocity disagree by $5 \cdot 10^7$ to $2 \cdot 10^8 \text{ ms}^{-1}$, whereas with the form used here they agree to the working precision of thirty digits. The author has, however, no derivation of it. It is an identification read off the closure condition, not a result, and its physical meaning is as little settled for the author as it will be for the reader; it is stated here as an open question rather than presented as understood. The second is the absence of a scale factor valid for time-dependent density parameters in a spatially curved spacetime; the present work has to fall back on the Λ CDM form, and a proper derivation would put the temporal part of this model on a firmer footing.

Use of Generative AI

The author used Claude (Anthropic), a large language model, in the preparation of this work. Its contribution comprised the translation of the manuscript into English and the subsequent language editing; the porting of the MathCad computation to Python using arbitrary-precision arithmetic; the independent numerical verification of

the cosmological algorithm and of the values reported in Sect. 6, including the mutual consistency of the closure relations given in the Appendix; and the generation of the figures, together with the drafting of the wording of Sects. 3 and 8 from the author's own documents (a worked solution and two short essays).

The underlying physical model, its derivations and its conclusions are the author's own. The author has reviewed every result obtained with this assistance and takes full responsibility for the content of this article. In accordance with the criteria for authorship, the language model is not listed as an author.

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