Analytical solution to the Hubble parameter

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Abstract

This article aims to show that the Hubble parameter is a time derivative of the Hubble constant. This is confirmed by three individual values from the empirical research:

Value 1: The Hubble parameter in the neighborhood of the earth; Value 2: The Hubble parameter at the cosmic horizon and value 3: The Hubble parameter at half the age of the universe.

It is developed an algorithm to calculate the Hubble parameter. With this algorithm, it is possible to make statements on the progress of the Hubble parameter in the future. On this way falling for the future of the Hubble parameter values are predicted.

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Introduction

1929 The American astrophysicist Edwin Powell Hubble published his observations to the expansion of the universe, which was postulated in 1927 by the Belgian priest Georges Lemaître. Hubble's work is the discovery of the proportionality of the flight of the galaxies to their distance from each other in shape $v = HR$ [1], Herein, v is the escape velocity and expansion of the space between galaxies, R is the distance between galaxies and H the Hubble constant, which he with the former agents from his observations to 500 Km / s Mpc noted.

To make this connection understandable for cosmology, ultimately applies to her that the universe looks the same everywhere and no excellent observer location or the center has[2] probably invented the German physicist Albert Einstein in his cosmology, the balloon model[3], In this the galaxies points to an expanding balloon surface or 3D sphere are. This has the disadvantage that it has a fourth spatial dimension, according to which the universe has a center point, although it is located in it and is not accessible for an observer in the sphere. Once this model has been studied by scientists, it turned out that no fourth spatial dimension is to be accommodated in our three-dimensional space. In addition, such a universe requires curved Riemann spaces. Such curves would have to be demonstrated. However, an experiment could not detect any curvature of space. In addition, an energy density for the universe, after which it is spatially flat resulted from the general theory of relativity. This density could verify scientists for the universe empirically.

The universe is not spatially curved. It is called "flat", which means that under consideration of cosmological a Euclidean space is present because the space is substantially empty. The balloon model had become obsolete and was the model of infinite raisin bread[4] replaced. In this the raisins represent the galaxies, and the rising (yeast) dough between them is for the expanding space. This model meets the basic assumptions of cosmology. There's no edge because it is infinite. In light of the immense size of the universe, to a diameter of 93 billion light years was calculated, one can certainly heard of *infinitely* speak.

The author of this essay imagines that this *raisin bread* at the beginning of the universe was very, very small, but essentially already looked like now, so the idea of a point-like compression is given all the matter before the Big Bang, and this point has only increased proportionally in the form of raisin bread.

In light of the importance of the expansion of the universe was seen in the last 90 years, ever deeper into space, not only, but also to determine the Hubble constant. Here it turned out that it decreases with a greater distance, which means that the Hubble constant was formerly smaller. Thus arose the term Hubble parameter, because with a look indeep space one looks into the past, because the speed of light is finite. One looked deeper than even celestial objects were visible, and found that the universe was opaque. They interpreted this nebula as the beginning of the universe, for its former size, especially a phase called *inflation* stands. She blew on the universe of a diameter less kilometers to many light years in no time.

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1 From "The Knaur, Universallexikon in 15 volumes," lexicographical Institute, Munich, 1992, p 2269
2 Source: The cosmological standard model, Matthias Bartel man, Springer-Verlag, 2019, p 5
3 Source: ff The Special and the General Theory of Relativity, Albert Einstein, Springer, 1988, pp 71st
4 https://scilogs.spektrum.de/relativ-einfach/kosmische-expansion-zwischen-gummiband-und-rosinenkuchen/

This phase does not affect the Hubble parameter. The Hubble parameter determines the phase of the accelerated expansion whose characteristic value it is.

The Hubble parameter

Hubble constant and Hubble parameters are (still) two different observations of the expansion of the universe. While the Hubble constant H_0 no relation to time has, although finally looked in the past, the Hubble parameter H has this very well. while H_0 refers to the distance to the cosmic horizon, that is the place where the escape velocity of celestial objects achieved by the expansion of space, the speed of light c and disappear so that the visibility, because their light can not reach us, describing the Hubble parameter the course of the expansion constants $H=v/R$ over the entire distance to the cosmic horizon - but after the time t that the light speed of light to reach us ($R=ct$).

Thus, even understandable why in cosmology in spheres of radius $R=c/H_0$ is thought, for greater our observation horizon can not be.

From the Hubble equation $v=HR$ It can be seen that the graph of v a rising straight line. Edwin Hubble has drawn the first diagram[5]:

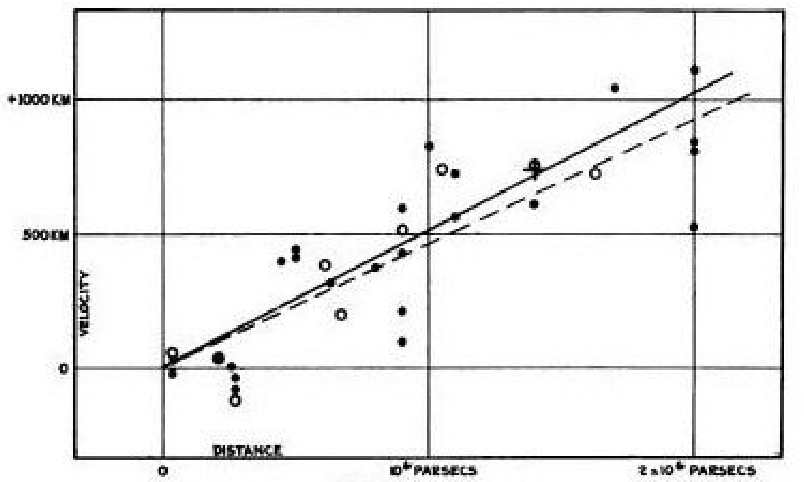


FIGURE 1

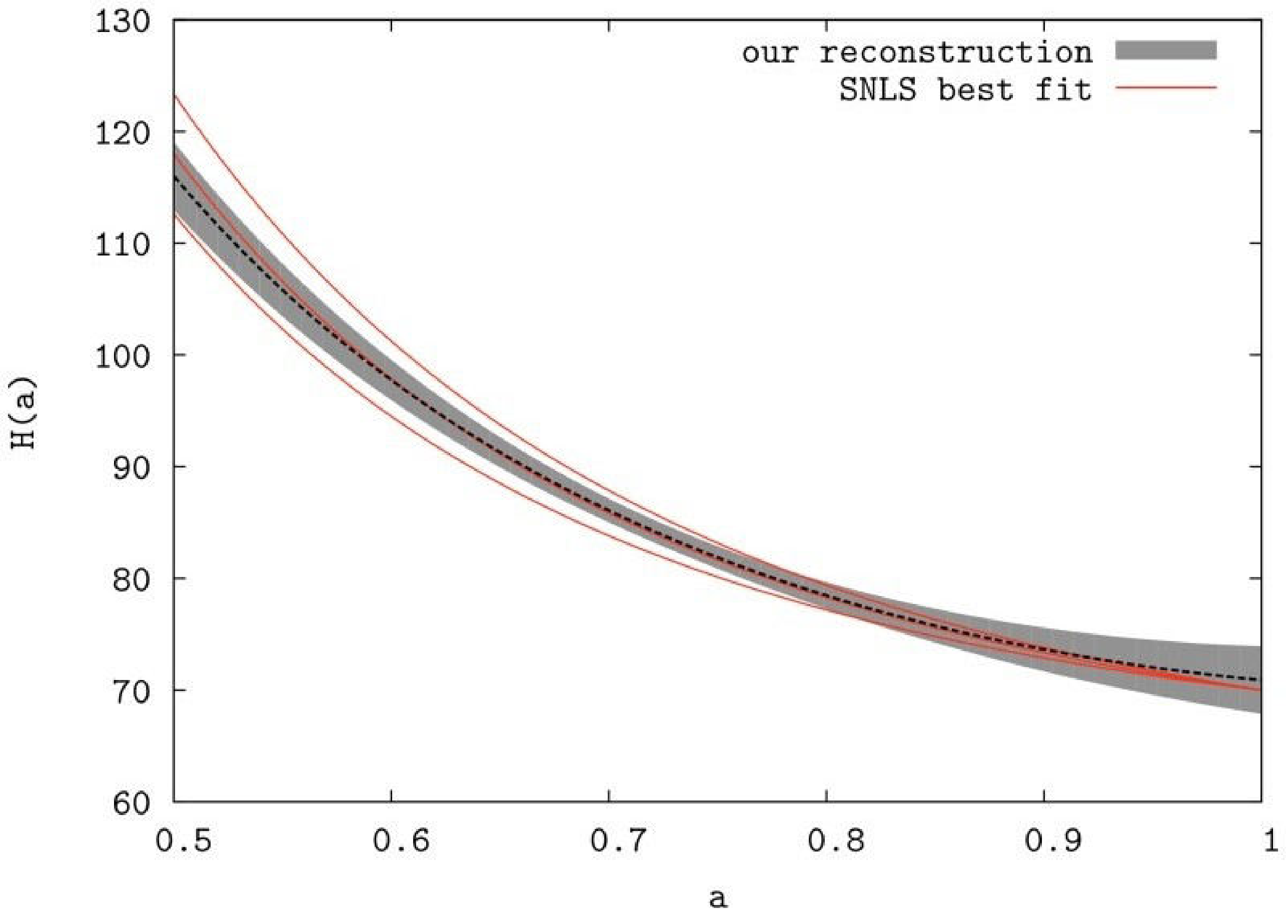
Velocity-Distance Relation among Extra-Galactic Nebulae.

Radial velocities, corrected for solar motion, are plotted against distances estimated from involved stars and mean luminosities of nebulae in a cluster. The black discs and full line represent the solution for solar motion using the nebulae individually; the circles and broken line represent the solution combining the nebulae into groups; the cross represents the mean velocity corresponding to the mean distance of 22 nebulae whose distances could not be estimated individually.

Source: Edwin Hubble, PNAS

Quite different the Hubble parameter whose chart I was kindly provided from his research Prof. Bartelmann:

5 Source: https://medium.com/@JChrHeuerAstro/edwin-hubble-und-die-expansion-des-weltalls-d14aa34ebf7d



However, the two representations of the expansion of the universe is your starting point mean. H_0 as well as H beginning with H_0 , H_0 remains constant over the entire time, while H grows towards the observer.

You can not see the graphs at first glance to their relationship. Only the fact that at the abscissa $H_0 = v/R$ the distance R, and wherein H is the time t in the form of the age of the universe a suggests that it could.

The exact value of H_0 is disputed for decades with different measurements. They are in the last 20 years all around the value H_0 = 70 km / s Mpc[6], Currently, a value of 74 in the conversation, which is causing a stir. Both values are in the following be considered here.

The Hubble parameter only superficially different from the constant H_0 s, because as from the Hubble connection $v = HR$ seen, the rate of expansion increases also with the distance, so that an expansion acceleration a (R) and a (t) the expansion velocity v in the following will be used.

Expansion acceleration a (R)

The basis of the model from the cosmological raisin bread is that the spatial geometry of the universe or "flat" is Euclidean over very long distances, the fact. General relativity Albert Einstein, which is the basis of this consideration, this holds a parameter before: the *critical density* ρ_c[7], It says

6 Source: https://de.wikipedia.org/wiki/Hubble-Konstante
7 Source: https://de.wikipedia.org/wiki/Dichteparameter

that the space of the universe is curved or not curved. Is the density to which also belongs energy as radiation, greater than ρ_c so the space is spherically curved. If it is smaller, it is hyperbolic, and has the density of the universe exactly the value of the *critical density s*o the space is flat or planar or Euclidean.

$$\rho_c = \frac{3 H_0^2}{8 \pi G} \rightarrow H_0 = \sqrt{\frac{8}{3} \pi \rho_c G}$$

This expression for H_0 must be developed. The approach to this is $a = 2MG/R^2$.

because $M = \frac{4}{3} \pi \rho_c R^3$ so that $a = \frac{8}{3} \pi \rho_c GR$,

It follows $a = H_0^2 R$

The approach $a = 2MG/R^2$ is remarkable. It is the doubled gravity-acceleration by Newton, and it is to repel. Not only that, the approach is very fit. The sealing parameters of dark energy ($\Omega_\Lambda = 0.685$) is also about twice as large as the density parameter of the baryonic + dark matter ($\Omega_M = 0.315$). Now, while the baryonic+dark matter is attractive, dark energy is repulsive. Following this approach, it does so twice as much, and it is she who is responsible for the expansion of the universe.

It seems as if the acceleration a expansive grow into the universe, which must not be confused with a growth of the universe at a constant density. In the gravitation in a solid sphere only the ball inside gravitationally acts as the model of gravity inside the earth[8] can be well understood. It is assumed here that this is also the case with the expansive force of dark energy.

Expansion velocity v (R)

To determine from the expansion acceleration a the expansion velocity v, it requires a differential equation: $a(R) = \frac{dv}{dt} = \frac{dv}{dR} \frac{dR}{dt} \rightarrow \frac{dR}{dt} = v \rightarrow \frac{dv}{dR} v = H_0^2 R$

The result is $v = \sqrt{c_1 + H_0^2 R^2} \rightarrow c_1 = v(R=0) = 0$

This is the Hubble relation: $v = H_0 R$, then that H_0 to $H_0 = \sqrt{\frac{8}{3} \pi \rho_c G}$ developed.

Terms after time: R (t), v (t), a (t)

As already mentioned, the Hubble parameter H is applied according to time. The idea is to determine Hubble's connection over time, and track from there to the many measurements of the galaxies and other celestial objects shining.

About the Hubble connection to get to $v = H_0 R \rightarrow \dot{R} = H_0 R \rightarrow \frac{dR}{dt} = H_0 R$

8 Source: Encyclopedia of Physics, lexicographical Institute, Munich, 1986, Vol 1, p 247

Differential equation: $\frac{dR}{R} = H_0 dt$

Integration on both sides: $\int \frac{1}{R} dR = \int H_0 dt \rightarrow \ln(R) + C_1 = H_0 t + C_2$

Summary of the constants of integration: $C_2 - C_1 = -C$

replacing of (- C) to ln (K): $\ln(R) = H_0 t + \ln(K)$

take e to: $R = e^{H_0 t + \ln(K)} \rightarrow R(t) = K e^{H_0 t}$

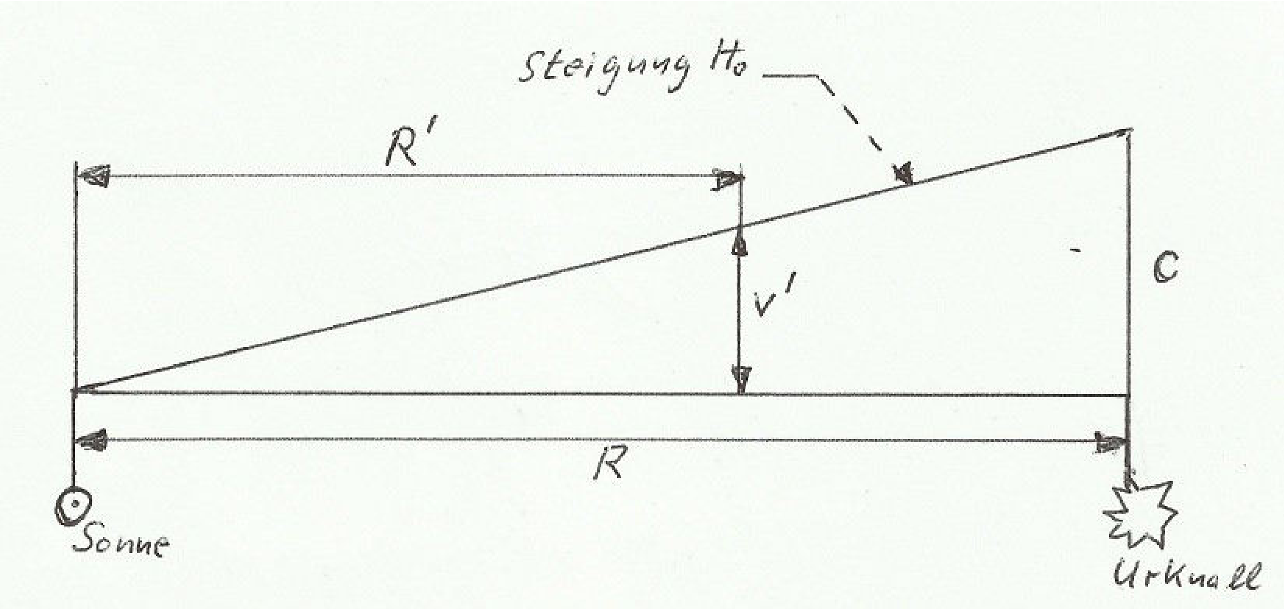
K is calculated on R (t = 0) = R_0 = 1[m], so that K = 1 [m].

There $v = \dot{R} = v(t) = K H_0 e^{H_0 t}$

and $a = \ddot{R} = a(t) = K H_0^2 e^{H_0 t}$

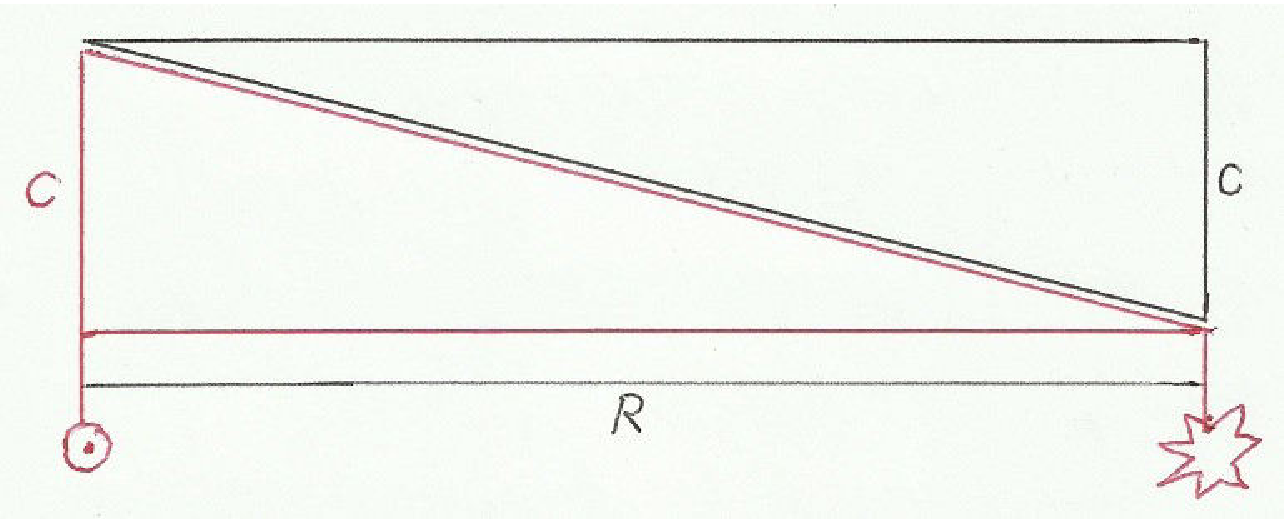
Probably the vast periods of time to be disconcerting. must be remembered, however, that for the beginning of the phase of expansion its initial radius must be considered the first the *inflation* presented. For details later.

The idea



The figure above shows the Hubble connection as usual. From the earth (or sun ʘ) of the distance R and the expansion speed v is measured. The slope of the velocity v' increase over the distance R' called H_0 is calculated according to the equation Hubble $H_0 = v'/R'$, Here there is equal to H_0 about c / R. Thus, the distance to the cosmic horizon near the Big Bang gives to $R = c/H_0$.

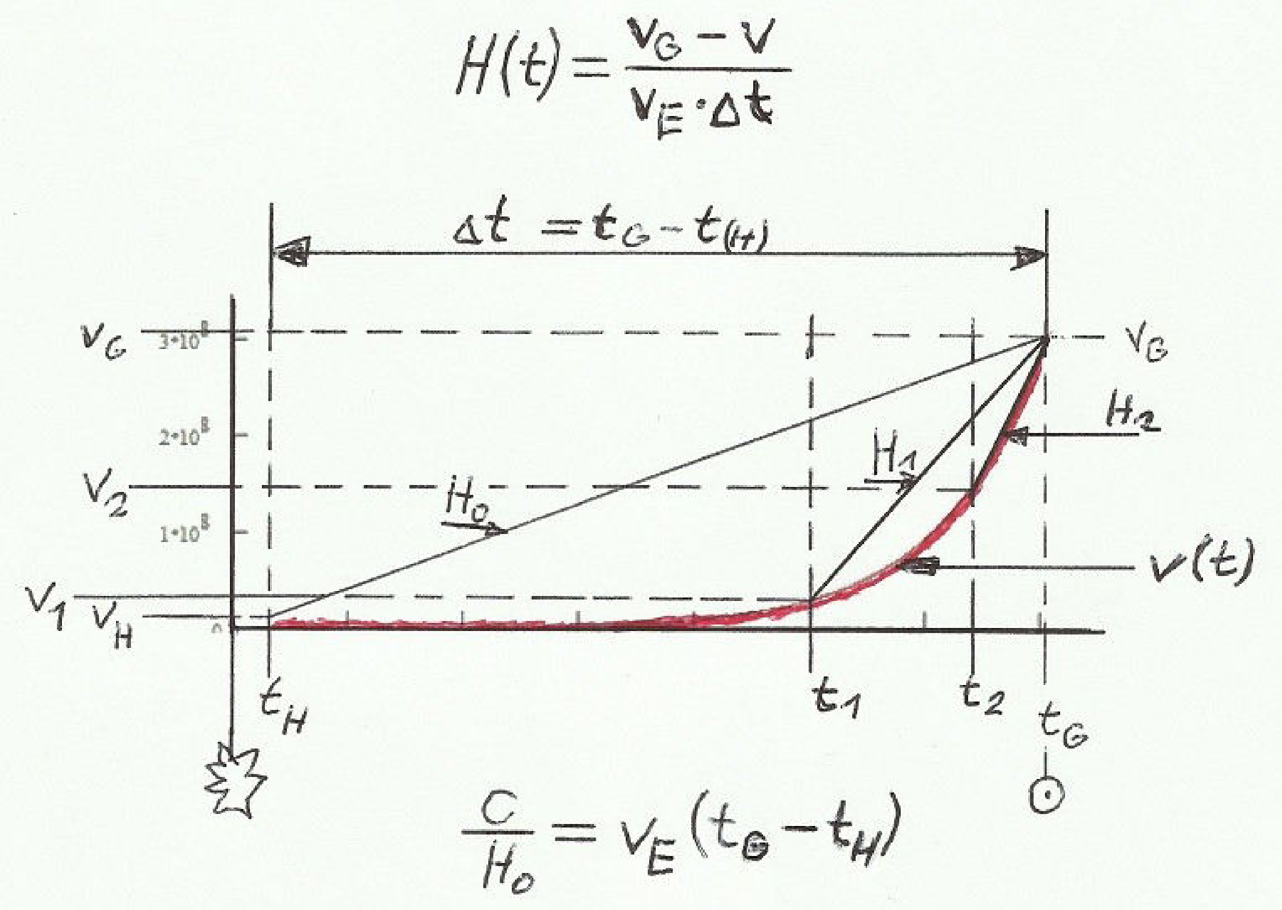
The core of the reconstruction of the Hubble parameter based on the Hubble constant but is an auxiliary construction:



The figure above shows the Hubble relation upside down. It is measured by the cosmic horizon, and calculates (red triangle). The fact that normally the distance R is in the past, leads to irritation. It is much closer,the expansion velocity of the cosmic horizon to grow in the future. The theory says Newton *actio est reactio*, making it all the same for the expansion as a thought experiment is whether is seen by the sun or the cosmic horizon. The flight or expansion rate is the same. The black triangle then shows the basis for calculating H's, so that looked back from the sun to the cosmic horizon.

In the standing figure below the course of the expansion velocity v does not look now more like a straight line from, but it should be noted and be shown on the final word that she still is on the distance R. Thus, the diagonal H by the above "rectangle" is as before, but just immanent.

In the calculation of the Hubble parameter H(t) the tide turns again to the conventional view, which is intended to illustrate that. Lying black over the red triangle in the above figure Hubble triangle. This is discussed in more detail in the following figure.



Here you can see the expansion velocity v(t) plotted as a red line after time. We, near the sun, already moving away with more than the speed of light from time zero, was known as the raisin bread one point. This expansion velocity v the author calls on the speed of the present t_G, The gradient H is now the ratio of △v and △R of the present and any one time / distance in the interval between the present and cosmic horizon. $\Delta v = v_G - v$ and $\Delta R = v_E(t_G - t)$,

The distance R occurs in this view, the expansion velocity with respect to time in the form of a speed and a distance difference $\Delta R = v_E \Delta t$ on. It is customary, here the speed of light c in the form of ct to take, but there is in this view the difference is that the increase in distance takes from time zero much longer up to the sun and presence, bridged as the light that distance. So must v with a replacement rate v_E be expected. For information follows.

From our observation point the sun from the expansion speeds △v result v as the difference between the speed v_G and the speed v at the location of the viewing or the distance (index 0, 1 and 2), measured in $\Delta R = v_E \Delta t$ Because during our observation, we are carried away with the expansion of space. The observed celestial body that is closer to the cosmic horizon is less carried away quickly, then that a speed difference △v shows that corresponds exactly to the expansion of space between us and the object being viewed.

Thus, distance and speed, ΔR and Δv, were clarified, making the modified Hubble relationship can be shown: $H(t) = \frac{\Delta v}{\Delta R} = \frac{v_G - v(t)}{v_E \Delta t}$

The cosmic horizon t_H

The difference between the rate of expansion to the present v_G and v the expansion speed at the cosmic horizon v_H is exactly the speed of light c. As a formula:

$$v_G - v_H = c \rightarrow K H_0 e^{H_0 t_G} - K H_0 e^{H_0 t_H}$$

From this it is the t_H of perceived time from time zero the expansion of the universe between the time zero and the cosmic horizon notice: $t_H = \frac{1}{H_0} \ln\left(\frac{K H_0 e^{H_0 t_G} - c}{K H_0} \right)$

The presence

The time of presence t_G is the time interval from time zero of the auxiliary construction until the present. He is iterated. As the characteristic value for this purpose is ideal Edwin Powell measurement of H = 500 [km / s Mpc] at a distance of one Megaparsec (MPC) from the earth.

The iteration is shown later.

The distance to the cosmic horizon c / H_0

If you look around from a point of observation from somewhere in space, we see essentially the same picture everywhere. In all directions you can look to a depth at which reaches the escape velocity of the celestial objects c. This is the horizon. It includes a ball in the center of the observer stands. In theoretical physics, this ball is often used as the basis of model calculations. Its radius is c/H_0. The distance between the presence of R_G and the cosmic horizon R_H automatically creates:

$$R_G - R_H = \frac{c}{H_0} \rightarrow K e^{H_0 t_G} - K e^{H_0 t_H} = \frac{c}{H_0}$$

As one can easily see, this is nothing more than the conversion of the formula from the chapter "The cosmic horizon tH"according to c/H_0. If t_H from t_G is calculated, this results in the distance c/H_0 by itself.

The replacement rate v_E

The age of the universe is a standard model of cosmology with a = 1 / H_0 specified. It turns out more than the speed of light. The time in which the light travels the distance between us, the observation and the cosmic horizon near the Big Bang is known as the age a.

As used herein auxiliary structure an exponential function of which is expansion rate $v(t) = K H_0 e^{H_0 t}$ used. The result is of course a value other than 1 / H_0 for a. Nevertheless, advances in v(t) here, the time continuously and uniformly progress, but at a different speed than the speed of light, namely, the replacement velocity v_E so that the distance of $R = c/H_0$ guaranteed even in the paced time Δt remains: $v_E = \frac{c}{H_0 (t_G - t_H)}$

The distance c/H_0 between us and the cosmic horizon time is simply divided by the time required for that route in the exponential function v(Δt).

A development v_E use after the time Δt about the distance function :

$K e^{H_0 t_G} - K e^{H_0 t} = c / H_0 = v_E \Delta t$ is incorrect because the distance about $R(t) = K e^{H_0 t}$ accelerated development. Finally $v_E (t_G - t) \neq K e^{H_0 t_G} - K e^{H_0 t}$ except the above context $\Delta t = t_G - t_H$,

It is in the Cape. Conclusion received on it again.

Time for the presence of t_G

In the presence of the Hubble parameter has initially the value of 500 [km / s Mpc]. Edwin Hubble found him 90 years ago, and, as much is being said, he has hardly changed since then in this theory. This value is the characteristic value of the iteration of the time t_G the present. Without him, the algorithm described previously does not make sense. The presence of t_G is our auxiliary construction in time much further after time zero as it does in reality, but the construction line must be finished thought.

The program of the iteration t_G use is as follows:

$$
\begin{aligned}
tG := \; & tG \leftarrow \frac{\ln\left(\frac{c}{K \cdot H0}\right)}{H0} + Y \\
& tH \leftarrow \frac{1}{H0} \cdot \ln\left[\frac{\left(K \cdot H0 \cdot e^{H0 \cdot tG} - c\right)}{K \cdot H0}\right] \\
& vE \leftarrow \frac{c}{H0 \cdot (tG - tH)} \\
& th \leftarrow \frac{Mpc}{vE} \\
& \text{while } \frac{K \cdot H0 \cdot e^{H0 \cdot tG} - K \cdot H0 \cdot e^{H0 \cdot (tG - th)}}{Mpc} \leq 500 \cdot \frac{1000}{Mpc} \\
& \quad tG \leftarrow tG - Y \cdot 10^{-6} \\
& \quad tH \leftarrow \frac{1}{H0} \cdot \ln\left[\frac{\left(K \cdot H0 \cdot e^{H0 \cdot tG} - c\right)}{K \cdot H0}\right] \\
& \quad vE \leftarrow \frac{c}{H0 \cdot (tG - tH)} \\
& \quad th \leftarrow \frac{Mpc}{vE} \\
& tG \leftarrow tG
\end{aligned}
$$

The Fig. Above is an excerpt of the calculation of my software and is therefore comprehension weaken for the casual reader. but it is quite easy to read when it is considered that it is common to read as from left to right and from top to bottom. Y is the time of one billion years in seconds.

First of all, start values must be determined:

Line 1: t_G is set to its lowest possible value. This occurs when the speed corresponds above all to present just the speed of light: $K H_0 e^{H_0 t_G} = c$

Then this is t_G increased generously by one billion years (+ Y), in order to ensure due to rounding errors of the software during the iteration no complex values occur.

Line 2: With line 1, the time interval of the cosmic horizon may from time zero t_H must be found (s. chap. The cosmic horizon tH, P 11).

Line 3: Here, the replacement rate is calculated (see chapter.. The replacement rate vE).

Line 4: t_h of the time interval of the Hubble Galaxy from the earth, by which E. Hubble the first measurement H's undertook (s cape is. The Hubble parameter), Namely, R = 1 Mpc. He is via the replacement v_E rate certainly.

Line 5: Here begins a while loop. The calculation contained therein is repeatedly performed until the condition is met. The condition is as follows: As long as the current to the t_G and th calculated Hubble parameter (s. Chap. The idea) is less than or equal to E. Hubble's H (500 Km / s Mpc) is further expected, because with decreasing t_G increases H. The calculation is in simplified terms $H(t)=\Delta v(t)/\Delta R(t)$. That the exponential function outputs also important consideration only with absolute values for the time the correct values. So has the value of the *Expansion velocity at the location of the object observed* the temporal distance are specified from time zero ($\Delta t=t_G-t_h$). Furthermore, the calculation on the right side shows the unit conversion of H of [Km / s / Mpc] to [m/s/m] or [s^{-1}], Because in this essay is all in SI units (s. Chap. Sizes used) Is calculated.

Line 6: After the comparison from line 5 is t_G decreased by 1,000 years. Concerning. the value of output at the final change 1,000 years H (t) hardly.

Line 7 to 9: The values for t_H, v_E and t_h be with the new t_G updated.

Line 10: After the end of the loop is determined, which value is outputted as a result - here it is t_G,

Values

Determining the time interval represented t_i

The fact that the exponential only absolute values accepted, this means values relative to the time zero, has the algorithm for determining the Hubble parameter (see p.9 above)

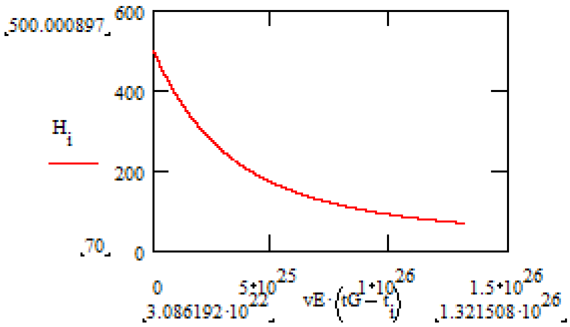
$$H(t)=\frac{K H_0 e^{H_0 t_G}-K H_0 e^{H_0 t_i}}{v_E(t_G-t_i)}$$

the temporal distance between the observed object and the time interval of the present are defined from time zero: $t_i=t_G-(t_G-t_H)\frac{i}{n}$

<u>Comment:</u> The software used here needs to create a graph individual values, the current with an index *i* are provided. The index can assume only natural numbers ranging from i = 0 ... n. Thus, the number of unique values is determined by n. Here n is specified, the index 1 indicates the distance to Hubble's celestial object in one Megaparsec distance. For this purpose, the distance c/H_0 to the

cosmic horizon divided by 1 Megaparsec, so that the distance as appears in n Megaparsec ($n = \frac{c}{H_0 Mpc}$) and is rounded to the whole number. This creates a slight inaccuracy.

H_0 = 70



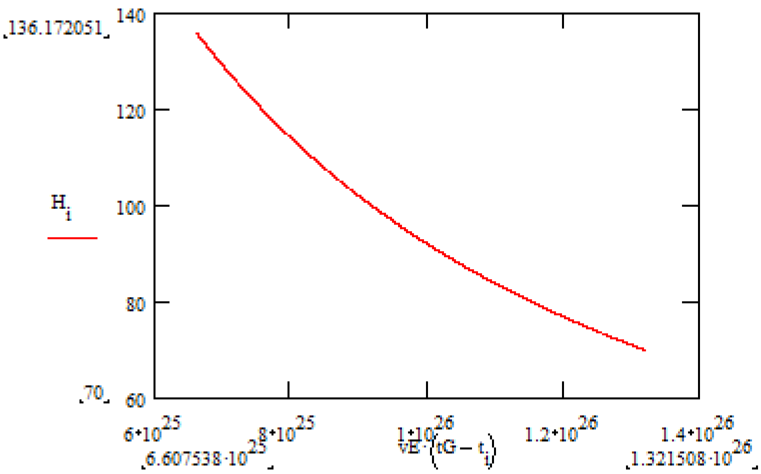
This is the history of the
Hubble parameter between the observer Earth (left) and the cosmic horizon (right).

The values given on the abscissa below show that the distance at the beginning of the graph as a preset Megaparsec is. The end of the graph shows the value of c/H_0 in meters, The final value of the ordinate (below) shows the Hubble constant, the initial value above the value measured by Hubble 1929 value of the Hubble parameter.

Compared to the measurement

In order to compare with the picture (see Fig. S.6 (!), Chap. The Hubble parameter) the Hubble parameter as it has been empirically determined, can only be viewed from the cosmic horizon from watched the first half of the Hubble parameter. For this, the default of t_i must be adjusted:

$$t_{i(0,5)} = t_H + \frac{t_G - t_H}{2} \frac{i}{n}$$

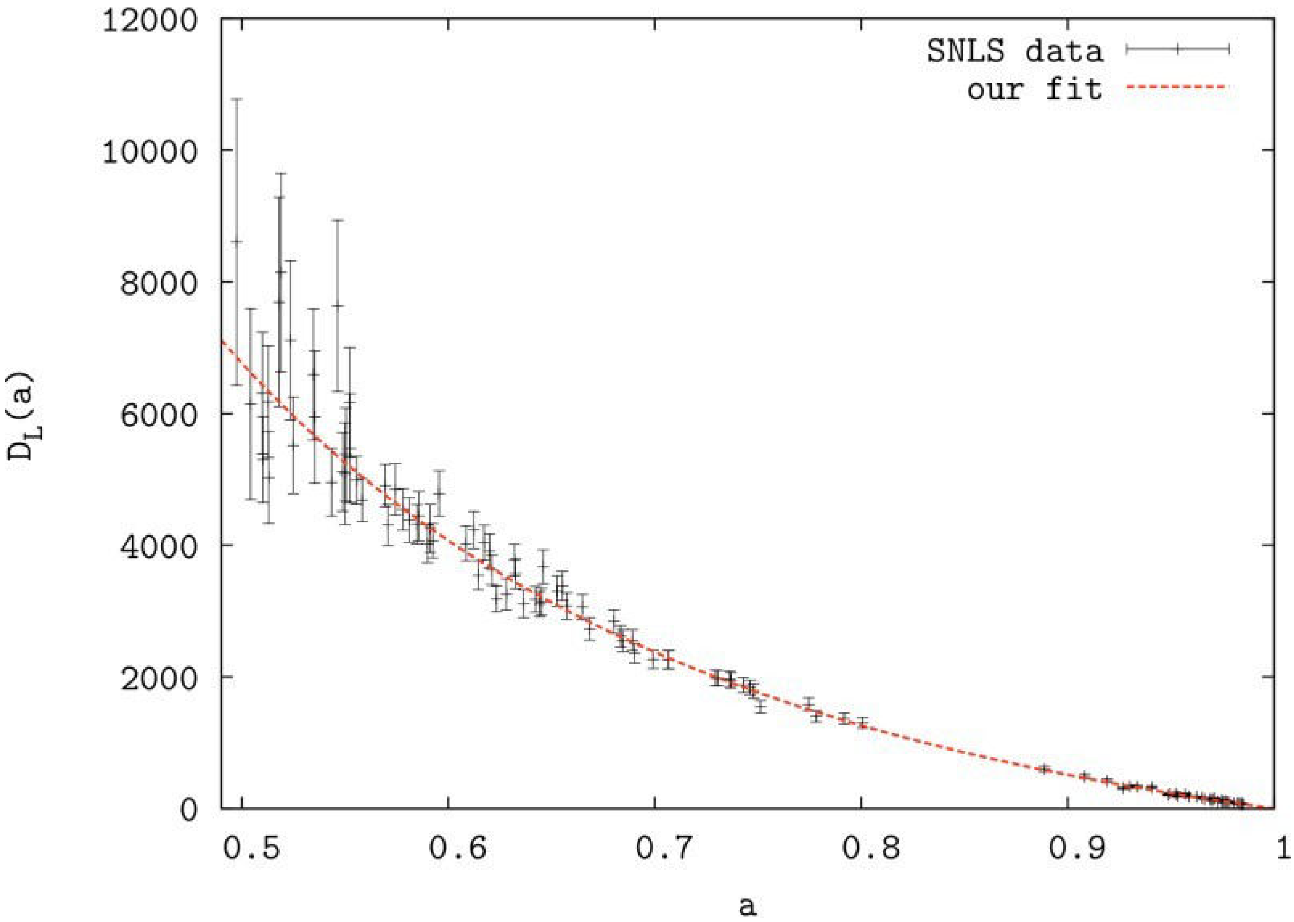


The figure reads as follows: One (top left) the abscissa provides links to supported the value of half the distance to the cosmic horizon, associated with the beginning of the graph right to bottom on the abscissa plotted the distance of cosmic horizon to us. On the ordinate down now, the value of the

Hubble constant, associated with the end of the graph (bottom right). Up on the ordinate finally the value of the Hubble Paramters exactly midway between the Earth and the cosmic horizon.

The second illustration in chapter. The Hubble parameter this shows about 117 ± 5 [km/s/Mpc], the reconstruction has ~ 136. Since the universe is a structure of the nature, it can cause such variations of H_0 come that cause deviations from the Hubble parameter from ideal, but the analysis is not yet over ... s. Cape.Walter Baade,

However, there are proper motions of the galaxies that distort the measurement result, as can be seen in the following figure, which the author also Prof. Dr. Bartelmann, University of Heidelberg, got provided:



The diagram shows the distribution of the luminosity distance D_L the measurements of flight-velocities of the galaxies according to their distance from the earth to the age of the universe.

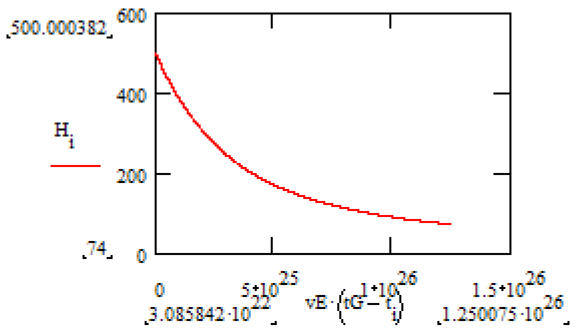
Quote Prof. Bartelmann: "The top diagram shows the luminosity distance as a function of Scale factor is now normalized to the to one. It shows how this distance grows by out- the universe and looking back in time. The luminosity distance is the square root of the ratio of luminosity and measured energy flow. "

Here could be seen the value of H that in $t_i = 0.5\,a$ could well be higher. Good is also apparent that the spread tends towards zero against the cosmic horizon, but that nothing about errors in the distance determination says (s. Chap.H0 = 74), as used in the following chapter, for the determination of the distance across the distance ladder[9] went wrong per se.

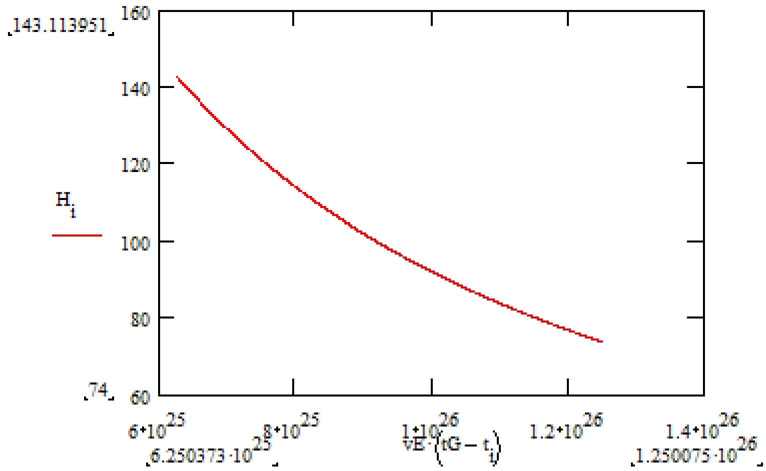
H_0 = 74

Recently the US have discovered astronomer and Nobel Prize winner Adam Riess and his colleagues found that the Hubble constant is perhaps higher than previously thought[10], Its value has a low tolerance of +/- 1.9% and is 74 [km/s/Mpc].

Besides the fact that this would be a disaster for the cosmological standard model, finally, the critical density ρ_c fits then no longer with the theoretical research agree that the universe *flat* would be there but an experiment[11] according to *flat* is, here is the graph will be shown to the Hubble parameter as if the space of the universe to be Euclidean also in theory in its geometry essentially:



and for the half Age:



The reading is congruent to the previous subsection.

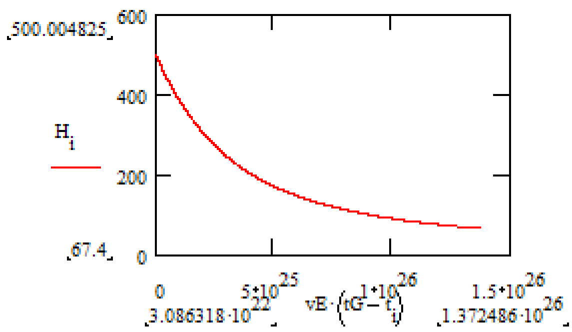
H_0= 67.4

The competitor with the Hubble constant by Adam Riess is the value of the Planck satellite ESA using the measurement of the cosmic background radiation was found[12]: H_0 = 67.4 [km/ s/Mpc][13],

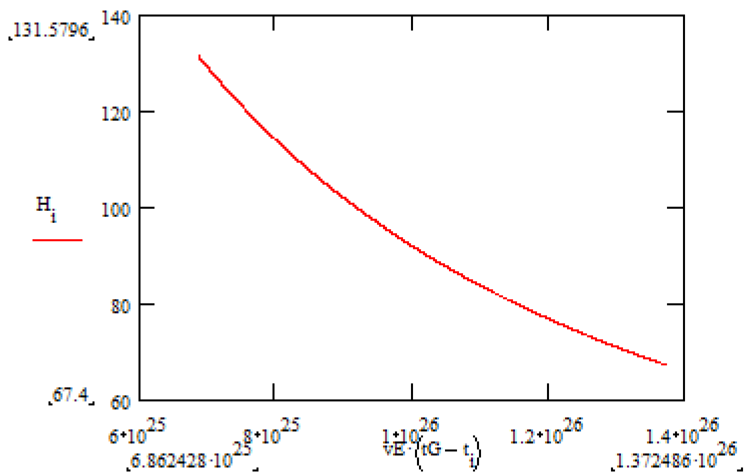
9 The astronomical distance ladder, Henri Wagner + Hendrik Tackenberg, Carl-Fuhlrott High School, 30.05.2018
10 Source: https://www.spektrum.de/news/hubbles-konstante-wird-immer-raetselhafter/1643430
11 Source: https://www.heise.de/tp/features/Das-Universum-ist-flach-3975407.html
12 Source: https://www.scinexx.de/news/kosmos/raetsel-um-kosmische-expansion-geht-weiter/
13 Source: https://sci.esa.int/web/planck/-/60504-measurements-of-the-hubble-constant

For him, the cosmological standard model is based. The critical density ρ_c with it's nearly equal to the empirically measured density of the universe, so to ρ_c from the density of luminous matter (Ω_b) Plus the dark matter (Ω_M- Ω_b), Dark energy (Ω_Λ) and the energy of the electromagnetic radiation (Ω_{rad}) Composed. Thus the geometry of the space of the universe is Euclidean in the theory of general relativity by Albert Einstein or *flat*,

In the following the proven diagrams:



And for comparison to measure the bottom half of the age a:



The value of H ($t_{i\,(0.5)}$) Is the comparison with the measurement (s. Chap. Compared to the measurement) Closer.

Walter Baade

But there is yet another way to improve the accuracy of the calculation: Edwin Hubble researcher with the 2.5-meter telescope on Mount Wilson[14]- far too inaccurate for this purpose instrument. In 1952, Walter Baade recognized[15], a German astronomer and astrophysicist who worked at Mount Wilson also since 1931, and carried out research with the new 5-meter telescope. He corrected "... the relationship between period and luminosity in the Cepheid variables and found that all those without the immense distances that Hubble had calculated doubled had." Source: footnote 14.

14 Source: https://www.spektrum.de/magazin/edwin-hubble-und-die-expansion-des-universums/821083
15 Source: https://de.wikipedia.org/wiki/Walter_Baade

Thus, t_h = 2Mpc / v_E, in addition, the Hubble constant Edwin Hubble (500 Km / s / Mpc) has at R = 1Mpc that eventually part of the Hubble parameters are re-calculated:

$v_{Hubble} = H_{Hubble}\ R_{Hubble}$

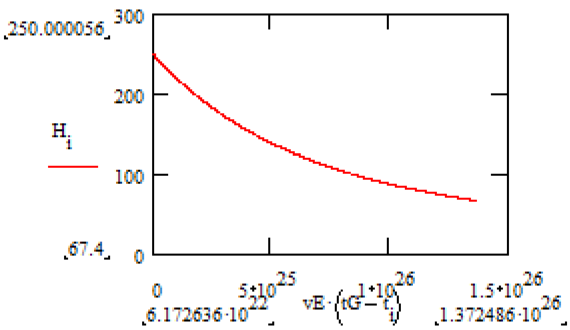
$H_{Baade} = v_{Hubble} / R_{Baade}$. $R_{Baade} = 2R_{Hubble}$ = 2Mpc

It follows H_{Baade} = 250 km/s/Mpc at R_{Baade} = 2Mpc.

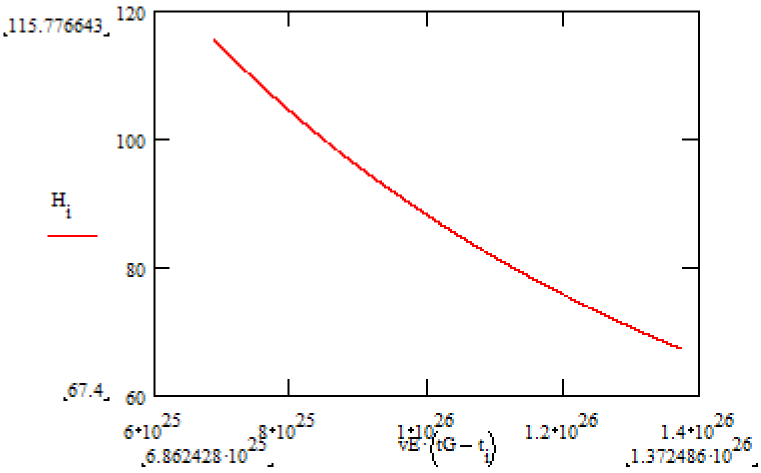
with H_0 = 67.4 [km/s/Mpc], at these values, the iteration of t_G (S. Chap. Time for the presence of tG) carried out. It looks like this:

$$
tG := \left|
\begin{array}{l}
tG \leftarrow \dfrac{\ln\left(\dfrac{c}{K \cdot H0}\right)}{H0} + Y \\[2ex]
tH \leftarrow \dfrac{1}{H0} \cdot \ln\left[\dfrac{\left(K \cdot H0 \cdot e^{H0 \cdot tG} - c\right)}{K \cdot H0}\right] \\[2ex]
vE \leftarrow \dfrac{c}{H0 \cdot (tG - tH)} \\[2ex]
th \leftarrow \dfrac{2 \cdot Mpc}{vE} \\[2ex]
\text{while } \dfrac{K \cdot H0 \cdot e^{H0 \cdot tG} - K \cdot H0 \cdot e^{H0 \cdot (tG - th)}}{2 \cdot Mpc} \le 250 \cdot \dfrac{1000}{Mpc} \\[2ex]
\quad \left|
\begin{array}{l}
tG \leftarrow tG - Y \cdot 10^{-6} \\[1ex]
tH \leftarrow \dfrac{1}{H0} \cdot \ln\left[\dfrac{\left(K \cdot H0 \cdot e^{H0 \cdot tG} - c\right)}{K \cdot H0}\right] \\[2ex]
vE \leftarrow \dfrac{c}{H0 \cdot (tG - tH)} \\[2ex]
th \leftarrow \dfrac{2 \cdot Mpc}{vE}
\end{array}
\right. \\[2ex]
tG \leftarrow tG
\end{array}
\right.
$$

The charts for the Hubble parameter H vary greatly. First, again H over almost the entire age of the universe:

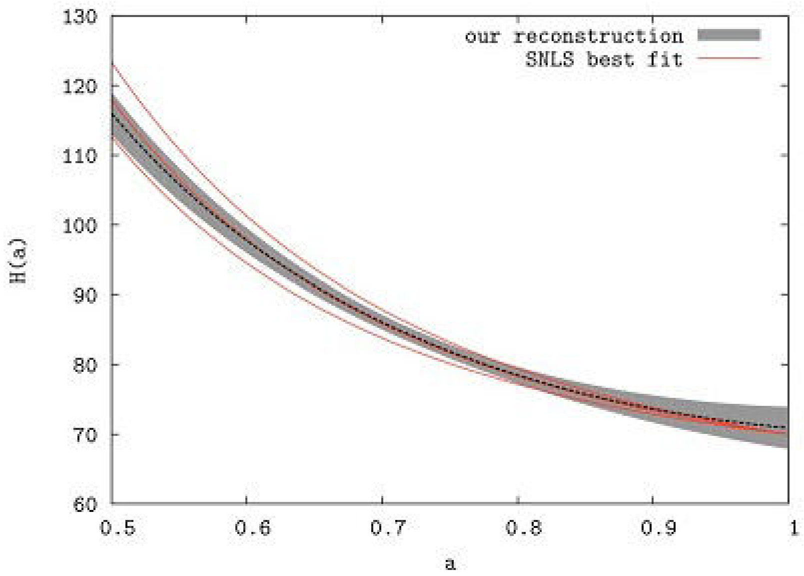


and then saw the chart H's for the period from cosmic horizon up to half the age of us:



Adam Riess' H_0 of 74 ± 1.9% (see FIG. Sec. H0 = 74) Returns here at the distance at half the age of the universe ~ 124 km/s/Mpc.

A comparison with the measurement effect:

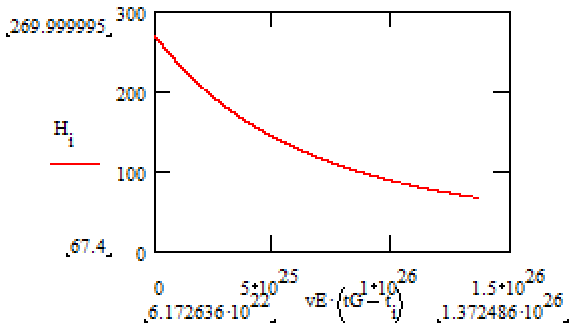


The value of H at half the age right now. Only in the curvature of the curve H's are still differences.

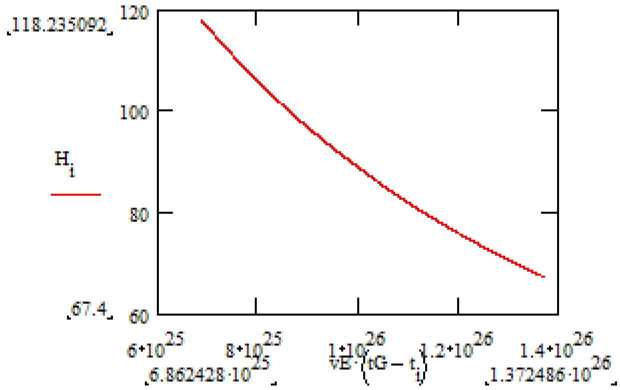
The California Institute of Technology is for Walter Baade's value of H_0 another value: 270 km / s / Mpc[16], The graphs fall even better:

a) Throughout the world Age:

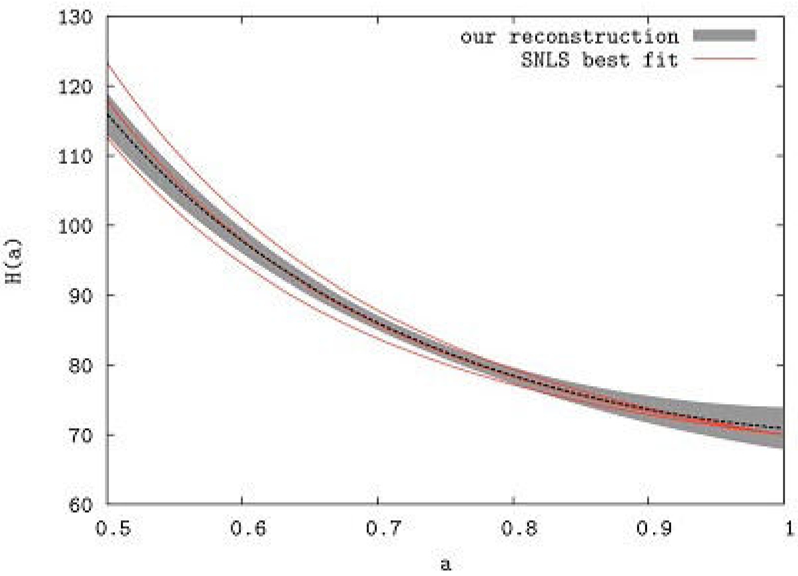
16 Source: https://ned.ipac.caltech.edu/level5/March03/Livio/Livio7.html



and b) About half the world age of the cosmic horizon:

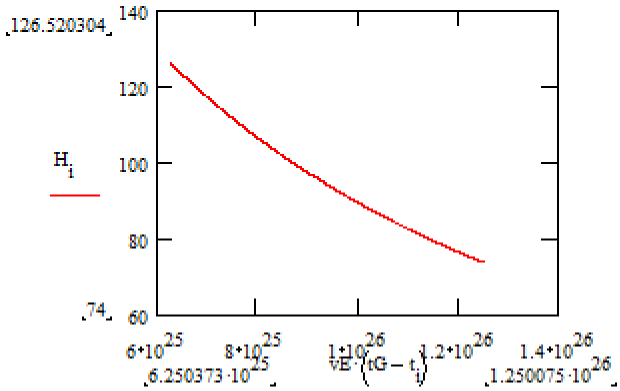


this once again the model:



The value at half the age of the world corresponds exactly to the original from the Supernova Legacy Survey (SNL).

Adam Riess' H_0 however, this results in



126.5 km/s/Mpc, which is at least ~ 3 km/s/Mpc out of tolerance from SNLS-model.

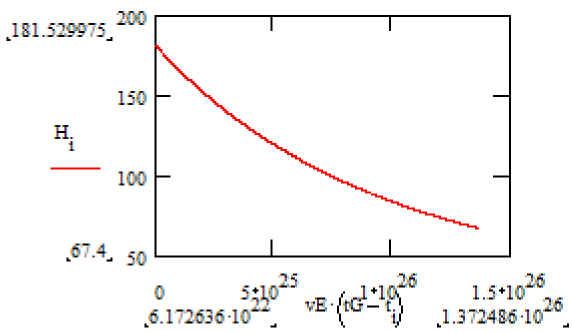
The future of the Hubble parameter

Ever since the Hubble parameter discovered spoken of accelerated expansion. Scientists suspect that the universe will explode one day that everything is torn apart by the expansion into its components. Therefore, the question is exciting as the Hubble parameter looks like in the future - Answer: He falls.

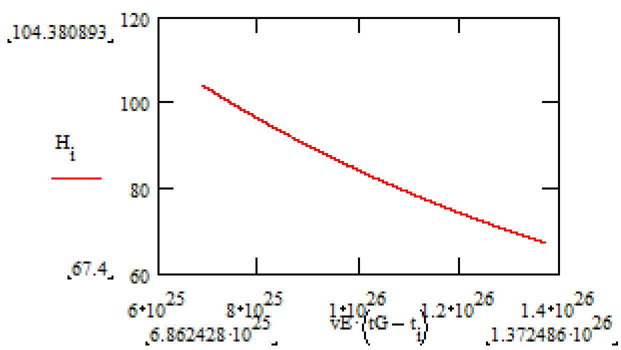
To verify this, the value is t_G increased on the basis of the previous chapter by one billion years old:

$$t_G{}' = t_G + Y$$

Starting from a H_0 of 67.4 [km / s / Mpc], all values are recalculated:



and the lower half:



In 10 million years ($t_G^{'}=t_G+Y \cdot 10^{-2}$) The value Baade's ~ 268 would be virtually unchanged and at half the age.

To bear in mind here is that the cosmic horizon is farther away from the Big Bang. Perhaps thus the universe in the cosmic horizon is not in a billion years opaque[17] as it is today.

With the calculation has shown that the future of the universe by no means ends in an explosion. Rather, linear develops after v (R) is still on the distance with H_0R. The Hubble parameter H(t) is only a monitor in time and space.

Before the phase of expansion

To determine how much of the *inflation* has on the size of the universe, has t_H be watched. Based on a H_0 of 67.4 the phase of expansion is obtained for the size of the universe at the time of the beginning t_H to which a radius is about, of $R(t_H)=\frac{K e^{H_0 t_H}}{c Y 10^{-3}} \approx 290$ Million light years.

This can be only accepted because the science for the time being an opacity determines exactly the cosmic horizon, the author associated started with the end of inflation phase. The universe would be through the *inflation* been blown up to a radius of ~ 290 million light years. The amount of time behind t_H is hidden for *inflation* but not significantly, because they had a much higher expansion parameters.

17 Information from Prof. Dr. Wolfgang J. Duschl, University of Kiel, received orally in 2009.

Conclusion

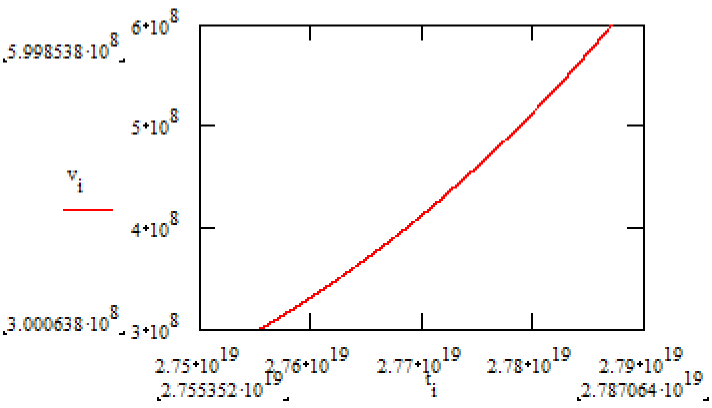
It could be shown that a derivative of the Hubble constant after the time maps the Hubble parameters approximately. It remains to be seen whether the deviation of the theory of the reality of the fact is enough that the universe is a natural phenomenon that is subject to natural fluctuations.

Noteworthy is that the theory predicts decreasing values for the future of the Hubble parameter, because that is provided by the end of the universe into question the dominant image. Reason for the decrease is the fact that the horizon at a given cosmic Hubble constant maintains a constant distance from the observer with the lapse of time; this while the expansion velocity on the basis of the Hubble relationship with the time increases steadily. The time interval of the theoretical calculation is therefore always shorter and the replacement velocity v_E increases. With increasing v_E decreases the time t_h, that is the distance to the observed celestial object in theory based on reducing the difference in the respective rates of expansion.

The question of the correctness, the Hubble connection as described in section "The idea" described observed on the head and subsequently rectify it in the calculation again, I will answer the end. To this end, I point out that even with the time derivative of the Hubble constant Hubble relation $H_0 = v/R$ is to allow:

Substituting in the equation for the Hubble parameter H (t) = △ v / ΔR (see p. 9 above the track difference) $v_E \Delta t$ by the denominator by the calculation $R(t) = K e^{H_0 t}$ so the Hubble parameter gives mathematically as Hubble constant: $H_0 = \dfrac{K H_0 e^{H_0 t_G} - K H_0 e^{H_0 t}}{K e^{H_0 t_G} - K e^{H_0 t}}$ - a mathematical identity[18].

Furthermore, v (t) shows with t_G+10 billion years, approximately the linear relationship H_0 use:



18 "[Identity] math. Expression that is valid for the entire domain of its variables; is in opp to an expression that is satisfied only for a certain value. for example, the*conditional equation* 3x = 9 meets only for x = 3, however, applies to the *I*[Dentitäts].-*Equation* $(A + b)^2 = a^2 + 2ab + b^2$for all values of a and b; one writes $(a + b)^2 \equiv a^2 + 2ab + b^2$ (≡ *I*[Dentitäts].-*Character*); ... " source: *The Knaur - Universal Encyclopedia in 15 volumes*, Lexicographical Institute, Munich, 1992, Volume 6, page 2313, keyword: identity, paragraph 5)

Attachment

Sizes used

Designation	**Symbol**	**Value**	**Unit**
Speed of Light	c	$2.9979 * 10^8$	m / s
Hubble constant	H_0	a) 67.4 Km/s/Mpc = $2.184*10^{-18}$ b) 74 Km/s/Mpc = $2.398*10^{-18}$	m/s/m
megaparsec	Mpc	$3.08656775 * 10^{22}$	m/Mpc
Age of the universe	a (1)	$1 / H_0$	- / -
gravitational constant	G	$6.674 * 10^{-11}$	$m^3/kg\ s^2$
Speed at the cosmic horizon	v_H	mutable	m/s
Speed to the present	v_G	mutable	m/s
Time on the cosmic horizon	t_H	mutable	s
The present time	t_G	mutable	s
Time interval measurement Hubble's	t_H	mutable	s
expansion rate	v	mutable	m / s
Distance at time zero	R	mutable	m
Hubble parameter	H	mutable	1 / s
replacement rate	v_e	mutable	m / s
A billion years in Seconds	Y	$10^9 * 365.24 * 24 * 60 * 60$	s
Time interval of the object observed from time zero	t	mutable	s
running Index	i	Natural numbers	- / -
largest Index	n	Natural numbers	- / -
Critical density after ART	ρ_c	$8.533 * 10^{-27}$ (In H_0= 67.4)	kg/m^3
Density parameter of matter incl. DM	Ω_M	0.315 in H_0 = 67.4 Km/s/Mpc	- / -
Density parameter of dark energy	Ω_Λ	0.685 in H_0 = 67.4 Km/s/Mpc	- / -
Distance from time zero for the presence	R_G	mutable	m
Distance from time zero to the cosmic horizon	R_H	mutable	m
expansion acceleration	a (2)	mutable	m / s^2
Circle number	π	3.14 ...	- / -
constant of integration	K	1	m
initial distance	R_0	1	m
light year	Lj	$9.460394 * 10^{15}$	m / Lj
luminosity distance	D_L	differently	- / -

Source Note

- Cape. Introduction: *The Knaur - Universal Encyclopedia in 15 volumes*, Lexicographical Institute, Munich, 1992, Volume 6, page 2269, keyword: Hubble, Edwin Powell
- Cape. Introduction: *The cosmological standard model - Basics, observations and limitations*, Matthias Bartelmann, Springer-Verlag, 2019, p 5
- Cape. Introduction: *The Special and the General Theory of Relativity,* Albert Einstein, Springer-Verlag, 23rd edition, 1988, reprint 2001, p 71 ff.
- Cape. Introduction: *Cosmic Expansion: rubber band or raisin cake*, Markus Pössel, 24.05.2009, https://scilogs.spektrum.de/relativ-einfach/kosmische-expansion-zwischen-gummiband-und-rosinenkuchen/
- Cape. Expansion acceleration a (R): *Encyclopedia of Physics*, Lexicographical Institute, Munich, 1986, Vol 1, p 247
- Fig Cape.. The Hubble parameter, P 5: *Realm of the Nebulae*, By EP Hubble. New Haven: Yale University Press, 1936. ISBN 9780300025002, location: NASA / ADS
- Cape. The Hubble parameter: *Hubble constant*. https://de.wikipedia.org/wiki/Hubble-Konstante
- Fig Cape.. The Hubble parameter, P 6: Prof. Dr. M. Bartelmann, Ruprecht-Karls-University of Heidelberg, about 2011
- Cape. Expansion acceleration a (R): *density parameter*, Wikipedia, https://de.wikipedia.org/wiki/Dichteparameter
- Cape. Compared to the measurement, P.17: Prof. Dr. M. Bartelmann, Ruprecht-Karls-University of Heidelberg, about 2011
- Cape. Compared to the measurement*The astronomical distance ladder - methods to determine the distances of stars,* Henri Wagner + Hendrik Tackenberg, Carl-Fuhlrott High School, Wuppertal, 30.05.2018, https://www.schuelerlabor-astronomie.de/wp-content/uploads/2018/08/Hendrik-Tackenberg-Henri-Wagner-Die-astronomische-Entfernungsleiter.pdf
- Cape. H0 = 74: *Hubble constant is always puzzling* Spectrum, 05.07.2019, https://www.spektrum.de/news/hubbles-konstante-wird-immer-raetselhafter/1643430
- Cape. H0 = 74: *The universe is flat*. https://www.heise.de/tp/features/Das-Universum-ist-flach-3975407.html , 25.02.2018
- Cape. H0= 67.4: *Measurements of the Hubble Constant*, ESA, 17.07.2018, https://sci.esa.int/web/planck/-/60504-measurements-of-the-hubble-constant
- Cape. H0= 67.4: *Mystery of cosmic expansion continues*, 27.01.2017, https://www.scinexx.de/news/kosmos/raetsel-um-kosmische-expansion-geht-weiter/

- Cape. Walter Baade: *Edwin Hubble and the expansion of the universe*, 01.09.1993, https://www.spektrum.de/magazin/edwin-hubble-und-die-expansion-des-universums/82108
- Cape. Walter Baade: *Walter Baade*, Wikipedia, https://de.wikipedia.org/wiki/Walter_Baade
- Cape. Walter Baade: *The World According to the Hubble Space Telescope*, Mario Livio, California Institute of Technology, https://ned.ipac.caltech.edu/level5/March03/Livio/Livio7.html
- Cape. Before the phase of expansion: C*onversation*, Prof. Dr. Wolfgang J. Duschl, University of Kiel, about 2009
- Cape. Conclusion: *The Knaur - Universal Encyclopedia in 15 volumes*, Lexicographical Institute, Munich, 1992, Volume 6, page 2313, keyword: identity, paragraph 5)

Software

- Operating system: Windows 10 Home Edition
- Text Editor: LibreOffice
- Formula Editors:
  - LibreOffice Writer
  - LibreOffice Math
- Mathematics: MathCad 7 Professional
- Graphic: Microsoft Paint
- browser: Mozilla Firefox
- PDF viewer: Foxit Reader
- Translator: https://www.onlinedoctranslator.com/de/translationprocess
- various software

Algorithm

$Mpc := 3.0856775 \cdot 10^{22}$ $H0 := 67.4 \cdot \frac{1000}{Mpc}$ $\Omega tot := 1.0005$ $G := 6.674 \cdot 10^{-11}$ $\rho c := \frac{3 \cdot H0^2}{8 \cdot \pi \cdot G}$ $\rho tot := \rho c \cdot \Omega tot$

$H0 = \sqrt{\frac{8}{3} \cdot \pi \cdot \rho c \cdot G}$ $c := 2.9979 \cdot 10^8$ $Y := 10^9 \cdot 365.24 \cdot 24 \cdot 60 \cdot 60$ $n := \text{floor}\left(\frac{c}{H0 \cdot Mpc}\right)$ $i := 0..n$ $K := 1$

$K \cdot H0 \cdot e^{H0 \cdot tG} = c$ $tG = \frac{\ln\left(\frac{c}{K \cdot H0}\right)}{H0}$

$K \cdot H0 \cdot e^{H0 \cdot tG} - K \cdot H0 \cdot e^{H0 \cdot tH} = c$ $tG = \frac{1}{H0} \cdot \ln\left(\frac{K \cdot H0 \cdot e^{H0 \cdot tG} - c}{K \cdot H0}\right)$

$K \cdot e^{H0 \cdot tG} - K \cdot e^{H0 \cdot tH} = \frac{c}{H0}$ $H1 := 270$

https://ned.ipac.caltech.edu/level5/March03/Livio/Livio7.html

$tG := tG \leftarrow tG + Y \cdot 10^{-0}$

$$
\begin{aligned}
tG := \; & tG \leftarrow \frac{\ln\left(\frac{c}{K \cdot H0}\right)}{H0} + Y \\
& tH \leftarrow \frac{1}{H0} \cdot \ln\left[\frac{\left(K \cdot H0 \cdot e^{H0 \cdot tG} - c\right)}{K \cdot H0}\right] \\
& vE \leftarrow \frac{c}{H0 \cdot (tG - tH)} \\
& th \leftarrow \frac{2 \cdot Mpc}{vE} \\
& \text{while } \frac{K \cdot H0 \cdot e^{H0 \cdot tG} - K \cdot H0 \cdot e^{H0 \cdot (tG - th)}}{2 \cdot Mpc} \le H1 \cdot \frac{1000}{Mpc} \\
& \quad tG \leftarrow tG - Y \cdot 10^{-6} \\
& \quad tH \leftarrow \frac{1}{H0} \cdot \ln\left[\frac{\left(K \cdot H0 \cdot e^{H0 \cdot tG} - c\right)}{K \cdot H0}\right] \\
& \quad vE \leftarrow \frac{c}{H0 \cdot (tG - tH)} \\
& \quad th \leftarrow \frac{2 \cdot Mpc}{vE} \\
& tG \leftarrow tG
\end{aligned}
$$

$tG = 2.7562 \cdot 10^{19}$

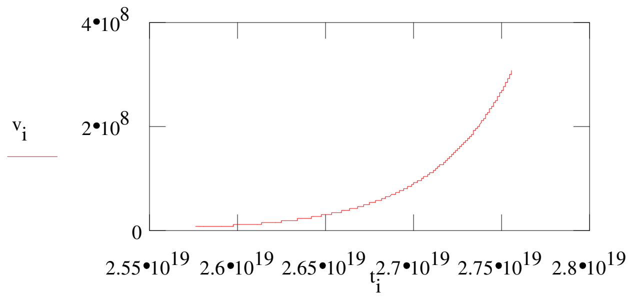
$tH := \frac{1}{H0} \cdot \ln\left(\frac{K \cdot H0 \cdot e^{H0 \cdot tG} - c}{K \cdot H0}\right)$

$tH = 2.5763 \cdot 10^{19}$

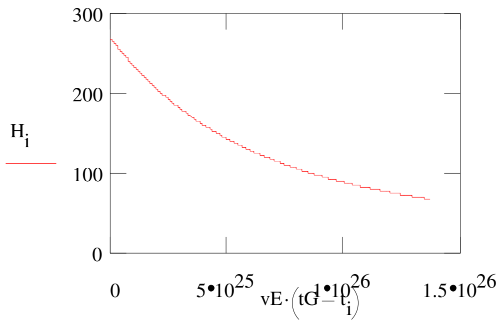
$vE := \frac{c}{H0 \cdot (tG - tH)}$

$vE = 7.627 \cdot 10^{7}$

$t_i := tG - (tG - tH) \cdot \frac{i}{n}$ $v_i := K \cdot H0 \cdot e^{H0 \cdot t_i}$

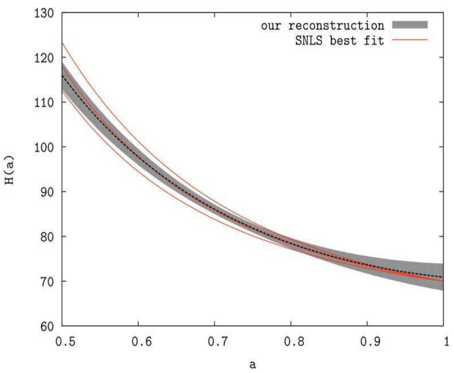


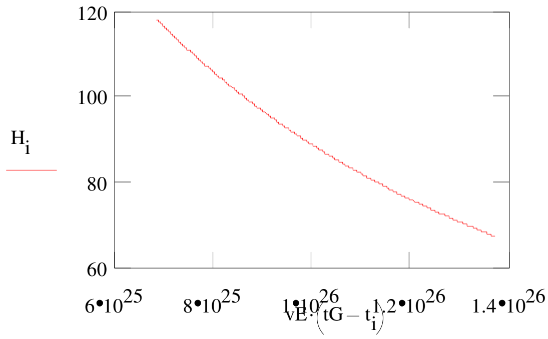
$$H_i := \frac{K \cdot H0 \cdot e^{H0 \cdot tG} - v_i}{vE \cdot (tG - t_i)} \cdot \frac{Mpc}{1000} \qquad i := 2 .. n$$



$$i := 0 .. n \qquad t_i := tH + \frac{tG - tH}{2} \cdot \frac{i}{n}$$

$$H_i := \frac{K \cdot H0 \cdot e^{H0 \cdot tG} - K \cdot H0 \cdot e^{H0 \cdot t_i}}{vE \cdot (tG - t_i)} \cdot \frac{Mpc}{1000}$$





$n = 4.447 \cdot 10^{3}$ *Mpc=[Mpc]

$RtH := \frac{K \cdot e^{H0 \cdot tH}}{c \cdot Y \cdot 10^{-3}}$ $RtH = 290.465$ Mio. [LJ]

$a2 := \frac{tG - tH}{Y}$ $a2 = 57.027$ Mrd. [Jahre]

$H0 \cdot 10^{10} = 2.184 \cdot 10^{-8}$ 1/s

$\rho c \cdot 10^{20} = 8.533 \cdot 10^{-7}$ kg/m³